Mechanics Mock Test

IJSO Theory Mock Test

Solutions

Problem 1 – Theme Park (15.00 points)

Part A – Getting to the Theme Park (1.00 points)

A1. What is the average velocity the tourist calculated, in km/h?

(0.20 points)

Calculation:

Flight duration based on the given time period: 22:30-08:00 = 14.5 hours

(0.10 points)

Distance between Colombo and Sydney = 8750 km

Therefore, the average velocity $= \frac{s}{t} = \frac{8750 \text{ km}}{14.5 \text{ h}}$ = **603.45 km/h**

(0.10 points)

$v = 603.45 \text{ km/h}$

A2. Using this new information, find the new average velocity of the plane

(0.50 points)

Calculation:

Take-off time in Sydney's time zone: 08:00 + 4.5 h = 12:30 PM

Actual flight duration: 22:30−12:30 = 10.0 h

(0.10 points)

Corrected average velocity: $v_{avg} = \frac{8750 \text{ km}}{10 \text{ h}}$ = **875 km/h**

(0.10 points)

$v = 875 \text{ km/h}$

A3. Which of the given graphs most accurately resembles the portion of the graph which shows the plane going from the time zone GMT+5 to the time zone GMT+6?

(0.30 points)

Answer : Graph I

(0.30 points)

Explanation : Because, when moving from the time zone GMT+5 to GMT+6 the distance will not shift therefore the distance will be continuous. So, we can eliminate options III and IV. When moving from GMT+5 to GMT+6, the local time will suddenly increase by 1 hour. Therefore, we can decide the correct option is option I.

Part B – Bungee Jumping (4.00 points)

B1. Find the values of x and θ.

(1.00 points)

Calculation:

Given that the man falls vertically for a distance x and that the elastic cord is only taut. We can find θ by applying basic trigonometry;

$$\sin\theta = \frac{20\text{ m}}{100\text{ m}} = 0.2$$

$$\theta = \sin^{-1} 0.2 = 11.54°$$

(0.40 points)

Then we can find x by using the Pythagorean theorem.

$$100^2 = 20^2 + (x + 10)^2$$

$$(x + 10) = 97.98\text{ m}$$

$$x = 87.98\text{ m}$$

(0.60 points)

$$x = 87.98\text{ m}$$

$$\theta = 11.54°$$

B2. Find the velocity of the man at the described moment.

(0.50 points)

Calculation:

Here since this is only vertical motion and since the elastic cord only becomes taut after that exact moment, we can find the velocity by using $v^2 = u^2 + 2as$

(0.20 points)

Since initial velocity is zero:

$v^2 = 2 \times 9.8 \times 87.98$

$v = 41.5\ ms^{-1}$

(0.30 points)

$v = 41.5\ ms^{-1}$

B3. Find the final velocity.

(0.50 points)

Calculation:

Initial kinetic energy: $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 75 \times 41.5^2 = 64584.4$ J

Final kinetic energy (after 10% is lost): $E_k = 64584.4 \times 0.90 \approx 58126$ J

(0.20 points)

Final velocity:$E_k = \frac{1}{2}m{v_f}^2$

$v_f = 39.4\ ms^{-1}$

(0.30 points)

$v_f = 39.4\ ms^{-1}$

B4. Find the maximum tension in the code.

(1.00 points)

Calculation:

The maximum tension in a pendulum is at its lowest point as the velocity at that point is the maximum which requires a higher centripetal force.

$$T = mg + \frac{mv_l}{L_0}$$

(0.20 points)

Total mechanical energy at the initial point in the pendulum:

$$E_{tot} = mgh + \frac{1}{2}mv_f{}^2$$

Here h is the height difference between the lowest point and the initial point.

$$E_{tot} = (75 \times 9.8 \times 2.02 + 58126)\ \mathrm{J} = 59610.7\ \mathrm{J}$$

(0.40 points)

Using the principle of conservation of mechanical energy:

$$E_{tot} = \frac{1}{2}mv_l{}^2 \rightarrow\ v_l{}^2 = 1589.62\ (\mathrm{ms}^{-1})^2$$

(0.20 points)

Therefore, the maximum tension:

$$T = mg + \frac{mv_l}{L_0} = 75 \times 9.8 + \frac{75 \times 1589.62}{100} \approx 1927.2\ \mathrm{N}$$

(0.20 points)

$$T = 1927.2\ \mathrm{N}$$

B5. Find the maximum extension of the code.

(1.00 points)

Calculation:

Let's calculate the spring constant of this cord:

$$k = \frac{YA}{L_0}$$

(0.35 points)

$$k = 35\ Nm^{-1}$$

Then, using Hooke's Law we can find the extension of the cord:

$$F = k\Delta L$$

(0.25 points)

Here the force is the maximum tension we found in sub-problem B4.

$$\Delta L = \frac{1927.2}{35}$$

$$\Delta L \approx 55.1\ m$$

(0.40 points)

$$\Delta L = 55.1\ m$$

Part C – Water Slide (4.00 points)

C1. Find the speed v_{launch} of the boat.

(0.50 points)

Calculation:

Using the conservation of energy:

$$\frac{1}{2}kx^2 = mgh + \frac{1}{2}m{v_i}^2$$

(0.30 points)

Here h_r is the height of the ramp (the vertical displacement).

$$\frac{1}{2} \times 1.60 \cdot 10^4 \times 0.8^2 = 80 \times 9.80 \times 1.50 + \frac{1}{2} \times 80 \times v_i^2$$

(0.15 points)

$v_i = 9.93\ \mathrm{ms^{-1}}$

(0.05 points)

$v_i = 9.93\ \mathrm{ms^{-1}}$

C2. Find the horizontal distance (range) R.

(0.75 points)

Calculation:

First, we find the initial vertical velocity:

$u = v_i \sin 20° = 3.4\ ms^{-1}$

(0.15 points)

Then, we find the period of flight considering the vertical components:

$$h = ut + \frac{1}{2}gt^2$$

Here the height (displacement) and gravitational acceleration is negative because we considered the initial vertical velocity positive.

$$-1.50 = 3.4t - 4.9t^2$$

Solving the quadratic equation we get: $t = 1.00$ s

(0.45 points)

Then we apply this time period to the horizontal motion to find the range:

$$R = v_i \cos 20° \times t = 9.33 \times 1.0 = 9.33\ m$$

(0.15 points)

$R = 9.33$ m

C3. Find the time of flight t and the vertical component of velocity on impact $v_{y,impact}$.

(0.50 points)

Calculation:

Time of flight (calculated in problem C2) = 1.00 s

(0.10 points)

Vertical component of velocity upon impact:

$v_{y,impact} = u + gt$

$v_{y,impact} = 3.4 - 9.8 = -6.40 \ ms^{-1}$

(0.40 points)

$t = 1.00 \ s$

$v_{y,impact} = -6.40 \ ms^{-1}$

C4. The pool is designed to begin 6.0 m horizontally from the ramp edge and extends for 6.0 m. Does the boat land in the pool?

(0.25 points)

Calculation:

9.33 - 6.0 = 3.33 m

3.33 < 6.0

Yes. The boat lands in the pool

(0.25 points)

C5. Find the range the boat can be compressed such that it still manages to land inside the pool

(2.00 points)

Calculation:

For the boat to land in the pool the range should be between 6m and 12m.

Therefore, we can find the initial launch velocities for the boat to land at a minimum distance of 6m and a maximum distance of 12m.

(0.10 points)

For range of 6m:

$$t = \frac{6}{v_1 \cos 20°}$$

Substituting this value for vertical motion:

$$h = v_1 \sin 20° \, t + \frac{1}{2} g t^2$$

$$-1.50 = v_1 \sin 20° \times \frac{6}{v_1 \cos 20°} - 4.9 \times \frac{36}{v_1^2 \cos^2 20°}$$

$$v_1^2 \approx 54.23$$

(0.60 points)

For range of 12m:

$$t = \frac{12}{v_2 \cos 20°}$$

Substituting this value for vertical motion:

$$h = v_2 \sin 20° \, t + \frac{1}{2} g t^2$$

$$-1.50 = v_2 \sin 20° \times \frac{12}{v_2 \cos 20°} - 4.9 \times \frac{144}{v_2^2 \cos^2 20°}$$

$$v_2^2 \approx 136.18$$

(0.60 points)

Using conservation of energy for both instances:

$$\frac{1}{2}kx_1^2 = mgh + \frac{1}{2}m{v_1}^2$$

$$x_1 = 0.647 \text{ m}$$

(0.30 points)

$$\frac{1}{2}kx_2^2 = mgh + \frac{1}{2}m{v_2}^2$$

$$x_2 = 0.911 \text{ m}$$

(0.30 points)

Therefore, range:

$$0.647 \text{ m} \leq x \leq 0.911 \text{ m}$$

(0.10 points)

Range: $0.647 \text{ m} \leq x \leq 0.911 \text{ m}$

Part D – Go-Karting (2.00 points)

D1. Calculate the maximum deceleration of the kart

(0.50 points)

Calculation:

$f_k = \mu_k N = \mu_k mg$

(0.20 points)

$$a_{max} = \frac{f_k}{m} = \mu_k g = 0.6 \cdot 9.8 = 5.88\ ms^{-2}$$

(0.30 points)

$$a_{max} = 5.88\ ms^{-2}$$

D2. Calculate the braking force the driver needs to apply in order to achieve the maximum deceleration.

(1.00 points)

Calculation:

First, find the total braking force needed at the wheels

$F = ma_{max} = 250 \times 5.88 = 1470\ N$

(0.20 points)

Force per wheel: $F_{wheel} = \frac{1470}{4} = 367.5\ N$

(0.20 points)

Hydraulic pressure: $P = A_2 F_{wheel} = \frac{367.5}{0.005} = 73500$ Pa

(0.30 points)

Force applied by the driver at the pedals:

$F_{pedal} = P \cdot A_1 = 73500 \times (2.0 \times 10^{-4}) = 14.7$ N

(0.30 points)

$F_{pedal} = 14.7$ N

D3. Determine the kinetic energy lost in the collision.

(0.50 points)

Calculation:

Converting the given speeds to ms^{-1} and applying the conservation of momentum

$mv_1 + mv_2 = (2m)v_f$

$v_f = 6.5\ ms^{-1}$

(0.15 points)

Initial Kinetic Energy:

$$\frac{1}{2} \times 250 \times 8.0^2 + \frac{1}{2} \times 250 \times 5.0^2 = 8000 + 3125 = 11125 \text{ J}$$

Final Kinetic Energy: $\frac{1}{2} \times 500 \times 6.5^2 = 10562.5$ J

Energy lost = 11125−10562.5 = 562.5 J

(0.35 points)

Energy lost = 562.5 J

Part E – Skateboarding (4.00 points)

E1. For an arbitrary natural number k, find the value of $\Delta H = H_k - H_{k+1}$

(0.80 points)

Calculation:

We can simply calculate this using energy conservation

$$Mg\,(H_k - H_{k+1}) = -\mu_k Mgd$$

(0.30 points)

$$(H_k - H_{k+1}) = -\mu_k d$$

$$\Delta H = -0.1 \times 1.5 = 0.15\text{ m}$$

(0.50 points)

$$\Delta H = 0.15\text{ m}$$

E2. How many times does the skater go up a ramp before he finally stops?

(0.70 points)

Calculation:

Here we can substitute H_n as 0 to find the instance at which the skater doesn't go up the ramp anymore.

$H_n = H_0 - n\Delta H$

$0 = 2.5 - 0.15\ n \rightarrow n = 16.67$

(0.40 points)

Since the answer is less than 17, the answer should be 16.

(0.30 points)

No. of times the skater goes up: 16

E3. After the skater stops, where does he stop?

(0.70 points)

Calculation:

In the 16th instance the skater reaches a height of: $H_{16} = 2.5 - 16 \times 0.15 = 0.10\ m$

(0.20 points)

Kinetic energy after the 16th instance: $E_{k16} = mg \times 0.1 = 78.4\ J$

The displacement of the skater along the flat plane: $E_{k16} = \mu_k Ns$

$s = 1\ m$

(0.50 points)

The skater stops 1m away from the foot of the right ramp along the flat plane.

E4. In which of the following cases does the result in E1 increase?

(0.35 points)

- ☐ Skater weighs M = 100kg
- ☐ Skater weighs M = 70kg
- ☐ Skate park is on the Moon (g = 1.62 $\frac{m}{s^2}$)
- ☐ Skate park is on Jupiter (g = 26.0 $\frac{m}{s^2}$)
- ☐ **Flat portion gets longer**
- ☐ Flat portion gets shorter
- ☐ **Friction on the inclines is not negligible**

Explanation:

Value in E1 is only dependent on the friction. Therefore, it will increase when the flat portion's distance increases and when the friction on the slopes is not negligible.

E5. What is his velocity at the lowest point?

(0.50 points)

Calculation:

Using the conservation of mechanical energy:

$$\frac{1}{2}Mv^2 = MgR$$

(0.20 points)

$$v = \sqrt{2gR} = 7.0\ ms^{-1}$$

(0.30 points)

$$v = 7.0\ ms^{-1}$$

E6. With what normal force is he acting on the track at the lowest point?

(0.65 points)

Calculation:

The normal force at the lowest point is his weight and the centripetal force.

(0.15 points)

$$N = Mg + \frac{Mv^2}{R} = 80 \times 9.8 + \frac{80 \times 7^2}{2.5} = 2352\ \text{N}$$

(0.50 points)

$$N = 2352\ \text{N}$$

E7. Find the pressure with which the skater pushes on the track at the lowest point.

(0.30 points)

Calculation:

Pressure at the lowest point:

$$P = \frac{N}{4A}$$

(0.15 points)

$$P = \frac{2352}{(4 \times 0.1 \div 10000)}$$

$$P = 5.88 \times 10^7\ \text{Pa}$$

(0.15 points)

$$P = 5.88 \times 10^7\ \text{Pa}$$

Extra Space for Problem 1:

Problem 2 – Mechanics of Atoms and Molecules (7.00 points)

Part A – The Bohr model of the Atom (1.60 points)

A1. Calculate the Bohr radius. Express it in angstroms ($1\text{Å} = 10^{-10}\text{m}$)

(0.30 points)

Calculation:

The Bohr radius is the value of R_n when n = 1.

(0.10 points)

$$a_0 = \frac{1^2 \times (6.62 \cdot 10^{-34})^2 \times 8.85 \cdot 10^{-12}}{\pi \times (9.1 \cdot 10^{-31}) \times (1.6 \cdot 10^{-19})^2} = 5.300 \cdot 10^{-11}\ \text{m} = 0.530\ \text{Å}$$

(0.20 points)

$a_0 = 0.530\ \text{Å}$

A2. Under this assumption, find the period of the orbit.

(0.80 points)

Calculation:

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \rightarrow v = \sqrt{\frac{GM}{r}}$$

(0.30 points)

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{GM/r}} = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(0.530 \cdot 10^{-10})^3}{6.67 \cdot 10^{-11} \times 1.6 \cdot 10^{-27}}} = 7420\ \text{s}$$

(0.50 points)

$T = 7420\ \text{s}$

A3. Using this new assumption, what is the period of the electron?

(0.50 points)

Calculation:

$$\frac{mv^2}{r} = F_e \rightarrow v = \sqrt{\frac{F_e r}{m}}$$

(0.20 points)

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{F_e \frac{r}{m}}} = 2\pi\sqrt{\frac{rm}{F_e}} = 2\pi\sqrt{\frac{0.530 \cdot 10^{-10} \times 9.1 \cdot 10^{-31}}{8.23 \cdot 10^{-8}}} = 1.52 \cdot 10^{-16}\ \mathrm{s}$$

(0.30 points)

$$T = 1.52 \cdot 10^{-16}\ \mathrm{s}$$

Part B – Rutherford's Experiment (1.50 points)

B1. What is the number density of gold atoms in a monoatomic gold layer

(0.50 points)

Calculation:

One atom occupies an area of

$$A = (2 \times 1.45 \cdot 10^{-10})^2 = 8.41 \cdot 10^{-20}$$

(0.20 points)

$$\text{Atom Density} = \frac{n}{A} = \frac{1}{8.41 \cdot 10^{-20}} = 1.19 \cdot 10^{19}$$

(0.30 points)

$$\text{Atom Density} = 1.19 \cdot 10^{19}$$

B2. What is the mass surface density ($\mathrm{mg/m^2}$) of monoatomic gold?

(0.50 points)

Calculation:

$\text{Mass density} = \mathrm{m_0} \times \text{Atom density}$

(0.20 points)

$\text{Mass density} = 3.27 \cdot 10^{-25} \times 1.19 \cdot 10^{19} = 3.89 \cdot 10^{-6}\ \mathrm{kg/m^2} = 3.89\ \mathrm{mg/m^2}$

(0.30 points)

$\text{Mass density} = 3.89\ \mathrm{mg/m^2}$

B3. Find the average thickness of the foil (in number of layers).

(0.50 points)

Calculation:

$$\frac{\sigma}{\text{mass density}} = \frac{7.776}{3.89} \approx 2$$

(0.50 points)

Number of layers = 2

Part C – Nuclear Density (2.20 points)

C1. Which of the following graphs would you expect to be a linear graph?

(0.25 points)

Since the density is constant, mass will be proportional to the volume which is proportional to the cube of r.

C2. Calculate the nuclear density for each of the given elements

(0.75 points)

Calculation:

$$\rho = \frac{M}{V} = \frac{A}{\frac{4}{3}\pi r^3} = \frac{3}{4\pi}\frac{A}{r^3}$$

(0.15 points)

Hydrogen: $\rho = \frac{3}{4\pi}\frac{1.66 \cdot 10^{-27}}{(0.85 \cdot 10^{-15})^3} = 6.45 \cdot 10^{17}\ \mathrm{kgm^{-3}}$

Helium: $\rho = \frac{3}{4\pi}\frac{6.64 \cdot 10^{-27}}{(1.35 \cdot 10^{-15})^3} = 6.44 \cdot 10^{17}\ \mathrm{kgm^{-3}}$

Lithium: $\rho = \frac{3}{4\pi}\frac{11.52 \cdot 10^{-27}}{(1.62 \cdot 10^{-15})^3} = 6.47 \cdot 10^{17}\ \mathrm{kgm^{-3}}$

Beryllium: $\rho = \frac{3}{4\pi}\frac{14.96 \cdot 10^{-27}}{(1.77 \cdot 10^{-15})^3} = 6.44 \cdot 10^{17}\ \mathrm{kgm^{-3}}$

(0.15x4 = 0.60 points)

C3. Calculate the mean value, the 4 errors and the mean error.

(0.75 points)

Calculation:

$$\text{Mean} = \frac{(6.45\ +\ 6.44\ +\ 6.47\ +\ 6.44)\ \cdot\ 10^{17}}{4} = 6.45 \cdot 10^{17}$$

(0.10 points)

$$\text{Error}_{\text{H}} = |6.45 \cdot 10^{17} - 6.45 \cdot 10^{17}| = 0.00 \cdot 10^{17}$$

$$\text{Error}_{\text{He}} = |6.45 \cdot 10^{17} - 6.44 \cdot 10^{17}| = 0.01 \cdot 10^{17}$$

$$\text{Error}_{\text{Li}} = |6.45 \cdot 10^{17} - 6.47 \cdot 10^{17}| = 0.02 \cdot 10^{17}$$

$$\text{Error}_{\text{Be}} = |6.45 \cdot 10^{17} - 6.44 \cdot 10^{17}| = 0.01 \cdot 10^{17}$$

(0.15x4 = 0.60 points)

$$\text{Mean error} = \frac{(0.00\ +\ 0.01\ +\ 0.02\ +\ 0.01\ )\ \cdot\ 10^{17}}{4} = 0.01 \cdot 10^{17}$$

(0.05 points)

C4. Calculate the value of the percent error.

(0.25 points)

Calculation:

$$\text{Percent error} = \frac{0.01\ \cdot\ 10^{17}}{6.45\ \cdot\ 10^{17}} \times 100\% = 0.155\ \%$$

C5. Based on your result in C4, is the hypothesis correct?

- ☐ **Yes**
- ☐ No

(0.20 points)

Part D – A look at the Simplest Molecules (1.70 points)

D1. Find the stiffness k of the bond.

(0.50 points)

Calculation:

The two forces are equal. Therefore,

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = kr$$

(0.20 points)

$$k = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^3} = \frac{1}{4\pi \times 8.85 \cdot 10^{-12}} \times \frac{(1.60 \cdot 10^{-19})^2}{(0.74 \cdot 10^{-10})^3} = 568.1 \text{ N/m}$$

(0.30 points)

$$k = 568.1 \text{ N/m}$$

D2. Find the frequency of its oscillations.

(0.70 points)

Calculation:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m^2}{2m} = \frac{m}{2}$$

(0.30 points)

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m/2}} = \frac{1}{2\pi}\sqrt{\frac{568.1}{(1.67 \cdot 10^{-27})/2}} = 1.31 \cdot 10^{14} \text{ Hz}$$

(0.40 points)

$$f = 1.31 \cdot 10^{14} \text{ Hz}$$

D3. Find the maximum bond length r_{max} and the minimum r_{min}

(0.30 points)

Calculation:

$r_{max} = 1.17r = 1.17 \times 0.74\ \text{Å} = 0.866\ \text{Å}$

$r_{min} = 0.83r = 0.83 \times 0.74\ \text{Å} = 0.614\ \text{Å}$

(0.15x2 = 0.30 points)

$r_{max} = 0.866\ \text{Å}$

$r_{min} = 0.614\ \text{Å}$

D4. If the oscillation energy of the hydrogen molecule is lowered and the amplitude of the oscillations decreases from 17% to 5%, what will happen to the frequency?

- ☐ It will get lower
- ☐ **It will stay the same**
- ☐ It will get higher

(0.20 points)

Explanation:

The frequency depends only on the reduced mass of the molecule and the stiffness. Therefore, the frequency will remain the same.

Extra Space for Problem 2:

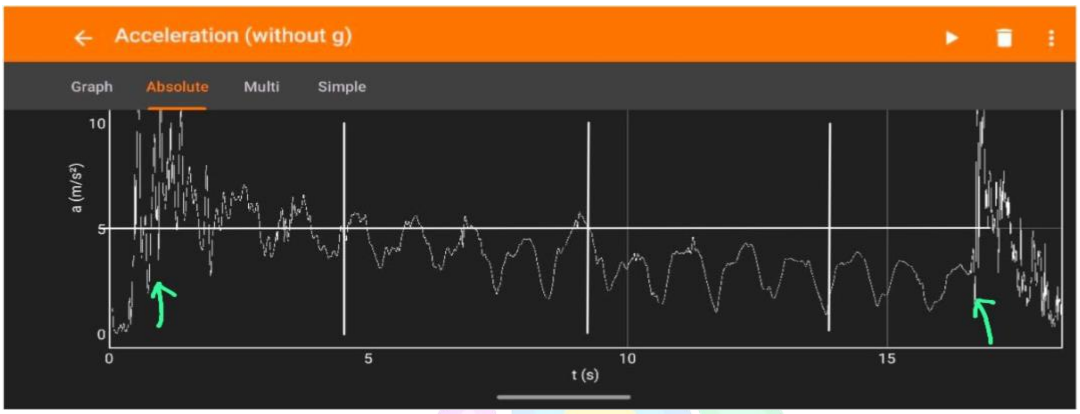
Problem 3 – A Simple Experiment (4.00 points)

Part A – Finding the Length of this Rope (2.20 points)

A1. Note the approximate moment at which the phone was released and the moment at which it was caught again to check the obtained data.

(0.30 points)

Calculation:



From the above diagram, you could see that the two moments are t=2 s and t=17 s.

A2. In which of the position is the acceleration maximum?

(0.25 points)

Calculation:

The accelerometer measures support force per mass, which is greatest at point B where the centripetal force is maximum.

A3. How many times does the phone pass through B during an entire period?

(0.20 points)

Calculation:

Twice—once while moving from A to C and once while moving back from C to A.

A4. Using the data in the graph, estimate the period of the studied pendulum.

(0.75 points)

Calculation:

When you count the number of maximum acceleration occurrences between t = 2 s and t = 17 s, you get 14 occurrences. This means, 7 periods have been completed. Therefore, the period is 15/14 = 1.07 seconds.

A5. Knowing the gravitational acceleration $g = 9.81\frac{m}{s^2}$, find the length of the rope.

(0.75 points)

Calculation:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(0.30 points)

$$l = (\frac{T}{2\pi})^2 g = (\frac{2.14}{2\pi})^2 \times 9.81 = 1.14 \text{ m}$$

(0.45 points)

$$l = 1.14 \text{ m}$$

A6. Why does the average acceleration overall decrease in time?

- ☐ Due to inaccuracies in the sensor of the phone
- ☐ **Due to air friction and similar dissipative forces**
- ☐ That is the normal behavior of an ideal gravitational pendulum

(0.25 points)

Part B – Elasticity of the Rope (1.80 points)

B1. Identify the moment at which the phone was launched upwards.

(0.15 points)

Calculation:

The large peak is when the phone was launched upwards. That peak occurs at 5.75s.

B2. Find the stiffness K of the spring.

(0.75 points)

Calculation:

The acceleration is approximately $75\ \mathrm{ms^{-2}}$.

(0.15 points)

$$\mathrm{ma} = \mathrm{kx}$$

(0.20 points)

$$\mathrm{k} = \frac{\mathrm{ma}}{\mathrm{x}} = \frac{0.197 \times 75}{0.1} = 147.75\ \mathrm{N/m}$$

(0.40 points)

$$\mathrm{k} = 147.75\ \mathrm{N/m}$$

B3. Find the relative elongation of the rope at that moment (the percent by which the rope gets longer)

(0.75 points)

Calculation:

The maximum acceleration is approximately $7.5\ \mathrm{ms^{-2}}$.

(0.15 points)

$$\mathrm{ma} = \mathrm{kx}$$

$$\mathrm{x} = \frac{\mathrm{ma}}{\mathrm{k}} = \frac{0.197\ \times\ 7.5}{147.75} = 0.01\ \mathrm{m} = 1\ \mathrm{cm}$$

(0.30 points)

Relative elongation = $\frac{0.01}{1} \times 100\ \% = 1\ \%$

(0.30 points)

B4. According to your result in part B3, is neglecting elasticity in part A reasonable?

(0.15 points)

Calculation:

The percentage error is 1% which is considered negligible in high-school experiments such as this one. Therefore, yes.

Extra Space for Problem 3:

Problem 4 – Meeting on the Vertical (4.00 points)

Part A – The Meeting (2.00 points)

A1. Determine the time t after launch at which the stones will meet.

(0.75 points)

Calculation:

We can write two equations of motion for each stone.

Stone A: $s_A = ut + \frac{1}{2}at^2 = \frac{1}{2}gt^2$

Stone B: $s_B = ut + \frac{1}{2}at^2 = 10t - \frac{1}{2}gt^2$

(0.50 points)

By adding those two equations together, we get

$s_A + s_B = 20\text{m} = 10t$

$t = 2\text{ s}$

(0.25 points)

$t = 2\text{ s}$

A2. At what height h (measured from the ground) will this happen?

(0.50 points)

Calculation:

$$s_B = 10 \times 2 - \frac{1}{2} \times 9.80 \times 2^2 = 0.4\text{ m}$$

A3. What condition must the initial speed v_0 satisfy for a collision to occur at all?

(0.75 points)

Calculation:

The initial velocity must be large enough for the collision to occur in the air and not after Stone B hits the ground.

The condition:

$$vt = 20$$

$$vt = \frac{1}{2}gt^2$$

(0.30 points)

$$\sqrt{\frac{40}{g}} = t = 2.02 \text{ s}$$

$$v = \frac{20}{t} = \frac{20}{2.02} = 9.90 \text{ ms}^{-1}$$

(0.30 points)

$$v > 9.90 \text{ ms}^{-1}$$

(0.15 points)

Condition : $v > 9.90 \text{ ms}^{-1}$

Part B – The Collision (2.00 points)

B1. Find the velocity of the stones after the collision. Is it directed upwards or downwards?

(0.50 points)

Calculation:

$$m_A v_A + m_B v_B = (m_A + m_B)\, v_f$$

(0.15 points)

$$m_A(-10) + 2m_A(7.5) = (m_A + 2m_A) v_f$$

$$v_f = 1.67\ \mathrm{ms^{-1}}$$

(0.30 points)

Since the sign is positive, this means upward direction.

(0.05 points)

$v_f = 1.67\ \mathrm{ms^{-1}}$

Direction : Upwards

B2. Find the maximum height that stone A reaches after it rebounds.

(1.00 points)

Calculation:

When the masses are the same in a 1D collision, the objects exchange velocities.

$v_{A,f} = v_{B,i} = 7.5\ ms^{-1}$

(0.20 points)

$\uparrow$ Direction: $v^2 = u^2 + 2as$

$$s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 7.5^2}{2 \times -9.80} = 2.87\ m$$

(0.80 points)

$s = 2.87\ m$

B3. Find the impulse (J or Δp) stone A exerts on stone B during the collision.

(0.50 points)

Calculation:

$J = \Delta p = m\,(v_f - v_i) = 1\,(-10 - 7.5) = -17.5\ Ns$

$J = \Delta p = -17.5\ Ns$

Extra Space for Problem 4: