Mechanics Mock Test No. 1

IJSO Theory Mock Test

This is an IJSO Mechanics mock test, designed to mimic the style, depth, and difficulty of chemistry questions found in the IJSO. Its aim is to help students strengthen their understanding of the physics concepts behind the IJSO and similar competitions.

The questions in this paper were made by the following past IJSO participants (in alphabetical order):

- Alex Jicu (Romania) – Mechanics Mock Test no. 1 Coordinator
- Luka Smiljanic (Serbia)
- Thenura Wickramaratna (Sri Lanka)

No.	Problem	Author	Marks
1	Theme Park	Thenura Wickramaratna Alex Jicu	15.00
2	Mechanics of Atoms and Molecules	Alex Jicu	7.00
3	A Simple Experiment	Alex Jicu	4.00
4	Meeting on the Vertical	Luka Smiljanic	4.00

In solving the questions, you might need to use the following constants:

Constant	Notation	Value
Acceleration due to gravity	g	$9.8\ \mathrm{ms^{-2}}$
Gravitational constant	G	$6.67 \cdot 10^{-11}\ \mathrm{m^3/\ kg \cdot s^2}$
Planck's constant	h	$6.62 \cdot 10^{-34}\ \mathrm{J \cdot s}$
Elementary charge	e	$1.6 \cdot 10^{-19}\ \mathrm{C}$
Speed of light in vacuum	c	$3 \cdot 10^{8}\ \mathrm{ms^{-1}}$
Density of water	ρ	$1000\ \mathrm{kg\ m^{-3}}$
Stefan-Boltzmann constant	σ	$5.67 \cdot 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m^2K^4}}$
Universal gas constant	R	$8.314\ \mathrm{J\ mol^{-1}\ K^{-1}}$ $0.0821\ \mathrm{atm\ L\ mol^{-1}\ K^{-1}}$
Avogadro's number	N_A	$6.022 \cdot 10^{23}\ \mathrm{mol^{-1}}$
Faraday's constant	F	$96\ 500\ \mathrm{C/mol}$
Pi	π	3.14
Electrical permittivity of free space	ε_0	$8.85 \cdot 10^{-12}\ \mathrm{F \cdot m^{-1}}$
Magnetic permeability of free space	μ_0	$4\pi \cdot 10^{-7}\ \mathrm{H/m}$
Mass of Earth		$5.97 \cdot 10^{24}\ \mathrm{kg}$
Mass of Moon		$7.35 \cdot 10^{22}\ \mathrm{kg}$
Mass of Sun		$1.99 \cdot 10^{30}\ \mathrm{kg}$
Radius of Earth		$6.4 \cdot 10^{6}\ \mathrm{km}$
Radius of Moon		$1.7 \cdot 10^{6}\ \mathrm{km}$
Radius of Sun		$6.96 \cdot 10^{8}\ \mathrm{km}$
Specific heat capacity of water	c_w	$4200\ \mathrm{J/kg \cdot {}^\circ C}$
Average molar mass of air	M	$28.9\ \mathrm{g/mol}$

If any other value is provided in the problem, use the value provided, not the one in the table. You can also use the following conversion formulas:

$T\ (\mathrm{K}) = t\ ({}^\circ\mathrm{C}) + 273$	$t\ ({}^\circ\mathrm{F}) = \frac{9}{5} t\ ({}^\circ\mathrm{C}) + 32$
$1\mathrm{bar} = 1\mathrm{atm} = 101\ 000\mathrm{Pa} = 760\mathrm{mmHg}$	$1\mathrm{u} = 1\mathrm{Da} = 1.66 \cdot 10^{-27}\mathrm{kg}$
$1\mathrm{L} = 10^{-3}\ \mathrm{m^3}$	$1\ \mathrm{day} = 24\mathrm{h}$

If needed, you can use the periodic table given bellow:

(Use atomic masses rounded to two decimal places.)

Problem 1 – Theme Park (15.00 points)

It's the year 2030. A newly built adventure theme park in Sydney (Australia) has quickly become the talk of the town. It features a wide range of attractions—from giant roller coasters to dizzying spinning rides, from haunted houses to futuristic VR games. But the centerpiece of the park, drawing thrill-seekers from all over, is its extreme-sports zone. Here, daring visitors can try free-fall towers, giant swings, and even bungee jumping from a high platform, secured only by a thick elastic cord.

Part A – Getting to the Theme Park (1.00 points)

The theme park quickly got the attention of tourists from all over the world, who are flying into Australia just to see it.

A Sri Lankan tourist flies from Colombo to Sidney to visit the theme park. His flight takes off at 8:00 AM (8:00) and lands at 10:30 PM (22:30). Curious about the average velocity of the plane, the tourist looked up the distance between Colombo and Sidney and found a value of 8750 km.

A1. What is the average velocity the tourist calculated, in km/h?

(0.20 points)

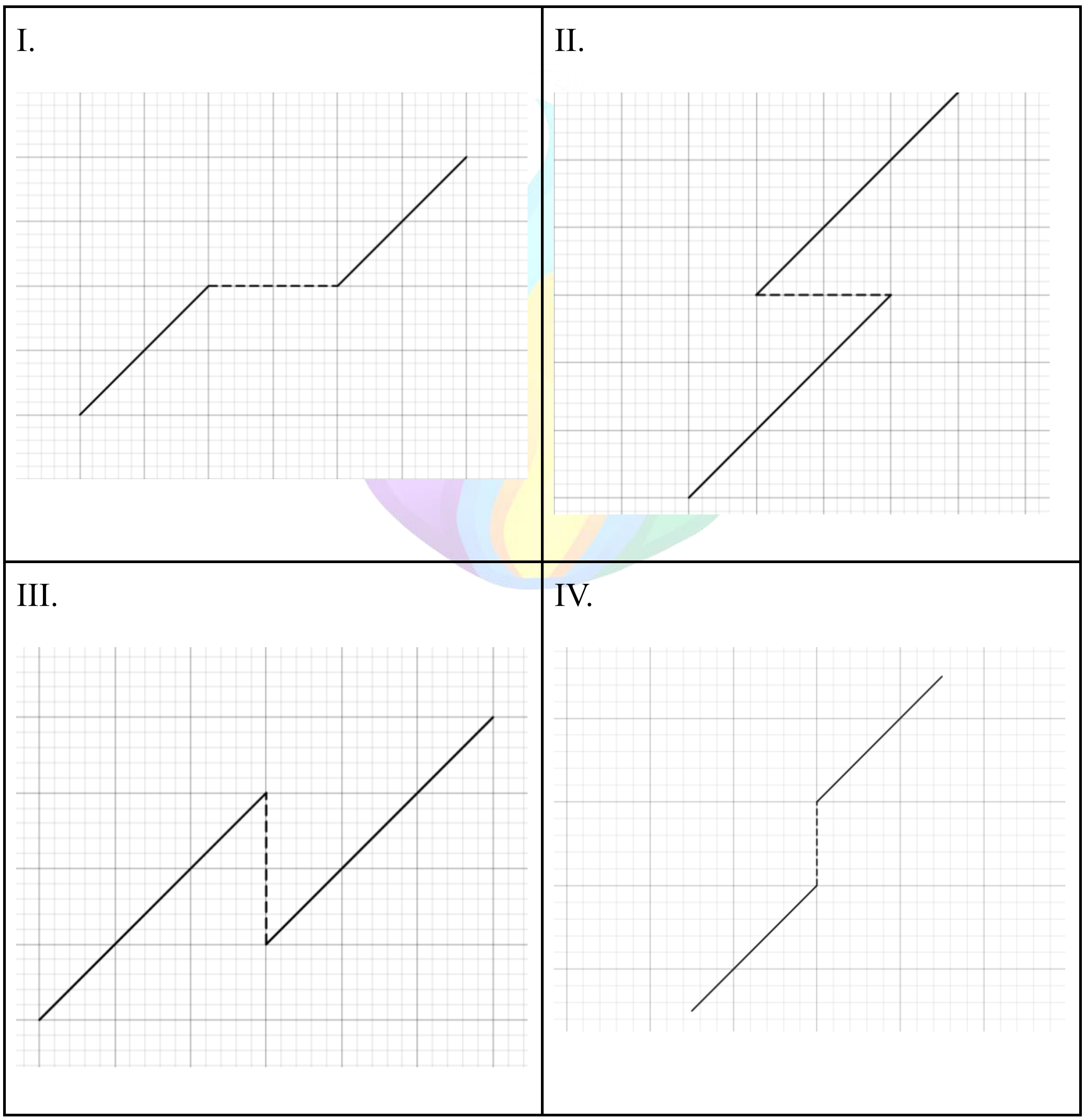
The result in A1 isn't correct, however, because the take-off and landing times are local times. The time zone in Colombo is GMT+5:30 (5 hours and 30 minutes ahead of the time in the British town Greenwich), while the time zone in Sidney is GMT+10 (10 hours ahead of the time in the British town Greenwich).

A2. Using this new information, find the new average velocity of the plane

(0.50 points)

Another curious tourist flying from Bucharest (the time zone is GMT+3) records the distance flown by the airplane as a function of the local time in the point the plane is currently in. Once he lands, after he gets to his hotel, he graphs the distance D as a function of the local time t.

A3. Which of the given graphs most accurately resembles the portion of the graph which shows the plane going from the time zone GMT+5 to the time zone GMT+6 (assume the plane moves with an approximately constant speed)?



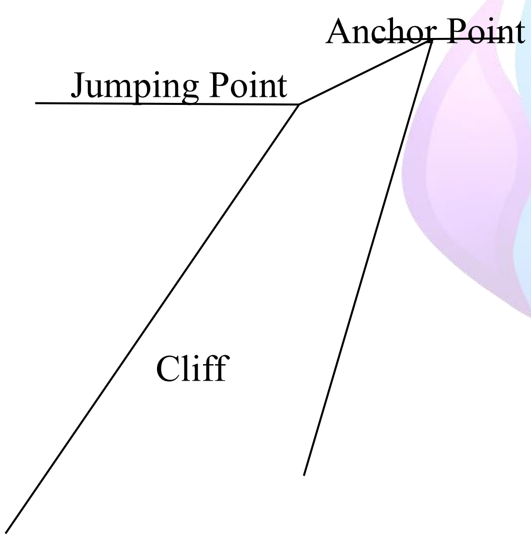
(0.30 points)

Part B – Bungee Jumping (4.00 points)

Bungee jumping, a very popular attraction of the theme park, is an extreme sport and recreational activity where a person jumps from a tall structure—such as a bridge, tower, or crane—while connected to a large elastic cord.

A man of mass m = 75 kg steps off a platform and jumps vertically downward while attached to a long elastic cord (natural length L_0 = 100 m).

It is said that the elastic cord's anchor point is located 20 m to the right and 10 m vertically above the jumping point. After falling vertically for a distance x from the platform, the cord is observed to make an angle of θ from the vertical; at this instant the cord just became taut, but hasn't started extending at all. Assume the anchor point is fixed and that air resistance is negligible.



B1. Find the values of x and θ.

(1.00 points)

B2. Find the velocity of the man at the described moment.

(0.50 points)

*If you weren't able to solve B2, take the velocity to be $100\ \mathrm{m\ s^{-1}}$.

After the initial vertical plunge, the jumper's cord is taut, and the jumper begins to swing like a pendulum about the anchor point. Treat the jumper as a point mass. Air resistance is negligible. For parts B3 and B4, consider the extension of the cord is much smaller than its relaxed length and therefore negligible.

B3. It is said that the person's energy is reduced by 10% at the moment the cord becomes taut and the remaining energy makes the person move in a direction perpendicular to the taut code. Find the final velocity.

(0.50 points)

B4. Find the maximum tension in the code.

(1.00 points)

B5. The young modulus of the cord is $5 \cdot 10^6$ Pa while the cross-section area is equal to $7 \cdot 10^{-4}$ m^2. Find the maximum extension of the code.

(1.00 points)

Part C – Water Slide (4.00 points)

A launch attraction in the park uses a compressed spring to send a small boat up a short ramp and then launch it through the air into a landing pool. Engineers want the boat to land safely in the pool every time.

Data:

- Mass of boat + rider: m = 80.0 kg.
- Spring constant: k = $1.60 \cdot 10^4$ Nm^{-1}.
- Spring compression before release: x = 0.800 m.
- The ramp rises to a vertical height h = 1.50 m above the spring base at angle $\theta = 20°$ above the horizontal axis.
- Assume the ramp and launch are frictionless and air resistance is negligible while the boat moves along the ramp and in flight.

C1. Find the speed v_{launch} of the boat immediately after it leaves the ramp (i.e., the speed when it becomes airborne).

(0.50 points)

C2. Find the horizontal distance (range) R from the end of the ramp to the point where the boat hits the water. (Treat the launch point as height h above the water surface.)

(0.75 points)

C3. Find the time of flight t and the vertical component of velocity on impact $v_{y,impact}$.

(0.50 points)

C4. The pool is designed to begin 6.0 m horizontally from the ramp edge and extends for 6.0 m. Does the boat land in the pool?

(0.25 points)

C5. Find the range the boat can be compressed such that it still manages to land inside the pool

(2.00 points)

Part D – Go-Karting (2.00 points)

A popular attraction in the theme park is the go-kart track. To ensure safety, each kart is equipped with a hydraulic braking system. When the driver presses the brake pedal, force is transmitted through brake fluid to the pistons at the wheels, generating friction to slow the kart down. Engineers want to ensure the karts can safely stop within 10 m when traveling at maximum speed.

Data:

- Mass of go-kart + driver: m = 250 kg
- Maximum speed: $v_{max} = 15 \text{ m s}^{-1}$
- Brake pedal piston area: $A_1 = 2.0 \cdot 10^{-4} \text{ m}^2$
- Wheel cylinder piston area: $A_2 = 5.0 \cdot 10^{-3} \text{ m}^2$
- Coefficient of kinetic friction between wheels and track: $\mu_k = 0.6$

Assumptions:

- The brake fluid is incompressible.
- The braking force is evenly distributed to all four wheels.

D1. Calculate the maximum deceleration of the kart

(0.50 points)

D2. Calculate the braking force the driver needs to apply in order to achieve the maximum deceleration.

(1.00 points)

D3. A go-cart travelling at a velocity of 28.8 km/h collides with another go-cart travelling at a velocity of 18.0 km/h in the same direction. The two go-karts then stick together. Determine the kinetic energy lost in the collision.

(0.50 points)

Part E – Skateboarding (4.00 points)

Another very popular attraction of the theme park is their skating park. There, they have a number of skating tracks, some of which we will analyze in this part. The first track is depicted in the picture:

The incline of both inclined planes is $\alpha = 30°$ and they are smooth (negligible friction), while the flat plane has a length of D = 1.5m and a coefficient of friction $\mu = 0.1$.

A skater (M = 80kg) goes down, from the ramp on the left, from a height of H_0 = 2.5m, and starts sliding freely. The maximum height that he reaches on the ramp on the right is H_1. After this, he goes back up the ramp on the left, at a maximum height H_2 and so on.

E1. For an arbitrary natural number k, find the value of $\Delta H = H_k - H_{k+1}$ (Hint: the value is independent of k)

(0.80 points)

It is known that the n-th height (the maximum height the n-th time he goes up a ramp) is $H_n = H_0 - n\Delta H$.

E2. How many times does the skater go up a ramp before he finally stops?

(0.70 points)

E3. After the skater stops, where does he stop?

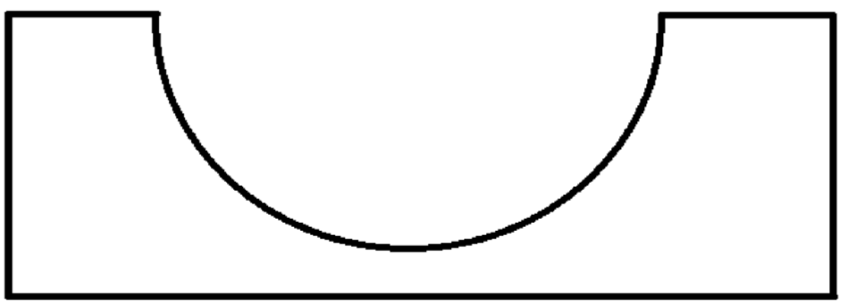
(0.70 points)

E4. In which of the following cases does the result in E1 increase?

- ☐ Skater weighs M = 100kg
- ☐ Skater weighs M = 70kg
- ☐ Skate Park is on the Moon (g = 1.62 $\frac{m}{s^2}$)
- ☐ Skate Park is on Jupiter (g = 26.0 $\frac{m}{s^2}$)
- ☐ Flat portion gets longer
- ☐ Flat portion gets shorter
- ☐ Friction on the inclines is not negligible

(0.35 points)

Another one of the tracks is the following one, which has the shape of a hemisphere of radius R = 2.5m. Its inside surface is smooth (no friction).



Now, the same skater as before, drops freely from the highest point on the ramp.

E5. What is his velocity at the lowest point?

(0.50 points)

E6. With what normal force is he acting on the track at the lowest point?

(0.65 points)

E7. Knowing that the area of one skateboard wheel that is pressed against the track is equal to $A = 0.1cm^2$, find the pressure with which the skater pushes on the track at the lowest point.

(0.30 points)

Problem 2 – Mechanics of Atoms and Molecules (7.00 points)

Although atomic physics is a complex topic, which we still don't fully understand and which requires a lot of mathematics, we can look at some simple atomic and molecular systems form the point of view of mechanics, because, in the end, all interactions can be studied as forces.

Part A – The Bohr model of the Atom (1.60 points)

The Bohr model of the atom models an atom as a small positively-charged nucleus, being orbited by an electron, which spins uniformly on a circular path. Moreover, the Danish physicist Niels Bohr, postulated that a quantity called the angular momentum of the electrons is quantized – that is, it can only have some values. Solving for the possible values of the radius of the orbit for a hydrogen atom, we get the equation:

$$R_n = \frac{n^2 h^2 \varepsilon_0}{\pi m_e e^2}$$

In the above formula, n is a non-zero natural number, $h = 6.62 \times 10^{-34} \mathrm{J \cdot s}$ is Planck's constant, $\varepsilon_0 = 8.85 \cdot 10^{-12} \mathrm{F/m}$ is the electrical permittivity of vacuum, $m_e = 9.1 \times 10^{-31} \mathrm{kg}$ is the mass of an electron and $e = 1.6 \times 10^{-19} \mathrm{C}$ is the elementary charge.

The Bohr radius (a_0) is defined as the smallest radius an electron's orbit in a hydrogen atom can have.

A1. Calculate the Bohr radius. Express it in angstroms ($1\text{Å} = 10^{-10}\mathrm{m}$)

(0.30 points)

From now on, if you couldn't solve A1, take $a_0 = 0.50\text{Å}$.

Let's now consider that the electron (found at the Bohr radius) only orbits under the influence of the gravitational attraction between it and the nucleus. The mass of the nucleus is $m_p = 1u = 1.6 \times 10^{-27}$kg

A2. Under this assumption, find the period of the orbit.

(0.80 points)

A way more realistic approach, considers that the rotation happens only under the influence of the electrostatic force between the electron and the nucleus. Its value is $F_e = 8.23 \times 10^{-8}$N.

A3. Using this new assumption, what is the period of the electron?

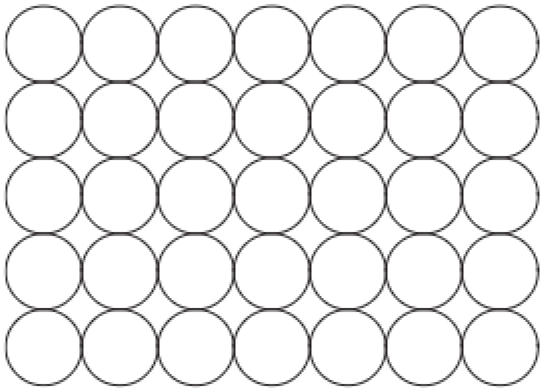
(0.50 points)

Part B – Rutherford's Experiment (1.50 points)

In the previous part, it was mentioned that electrons orbit the nucleus which is concentrated in the middle of the atom. This is the basis of the Rutherford model of the atom (which does not include the quantization of angular momentum), which was built after the famous Rutherford scattering experiments in the early 1900s.

In his work, Rutherford is said to have used a monoatomic gold layer. For this problem, let us consider gold atoms to be spheres of radius $r = 1.45 \times 10^{-10}$m and mass $m_0 = 196.97u = 3.27 \times 10^{-25}$kg.

In a monoatomic gold foil, the gold atoms are arranged in a square-packing model (see the figure below).



B1. Using the model provided, what is the number density of gold atoms in a monoatomic gold layer (gold atoms/meter squared)?

(0.50 points)

B2. What is the mass surface density ($\mathrm{mg/m^2}$) of monoatomic gold?

(0.50 points)

Rutherford's gold foil wasn't a perfectly monoatomic layer.

B3. If the real surface density of the foil Rutherford used is $\sigma = 7.776\ \mathrm{mg/m^2}$, find the average thickness of the foil (in number of layers).

(0.50 points)

Part C – Nuclear Density (2.20 points)

Let's consider the nuclei of atoms to be very small sphere, of radius r and mass A. In this part we'll check the following hypothesis:

"All nuclear matter is of the same nature, and very similar in structure and composition, which will lead to an almost constant nuclear density ρ"

C1. If the hypothesis were true, which of the following graphs would you expect to be a linear graph?

- ☐ A in terms of r^3
- ☐ A in terms of r^2
- ☐ A in terms of r^1
- ☐ A in terms of r^{-1}
- ☐ A in terms of r^{-2}
- ☐ A in terms of r^{-3}

(0.25 points)

Because nuclei are very small, we measure nuclear radii in femtometers (fm). The prefix "femto-" is equivalent to 10^{-15}. The following table with nuclear radii and nuclear masses is given:

Atom	Hydrogen	Helium	Lithium	Beryllium
r (fm)	0.85	1.35	1.62	1.77
A (10^{-27}kg)	1.66	6.64	11.52	14.96

C2. Calculate the nuclear density for each of the given elements

(0.75 points)

C3. Calculate the mean value, the 4 errors and the mean error.

(0.75 points)

C4. Calculate the value of the percent error.

(0.25 points)

C5. Based on your result in C4, is the hypothesis correct?

- ☐ Yes
- ☐ No

(0.20 points)

Part D – A look at the Simplest Molecules (1.70 points)

When two atoms bond, they form a diatomic molecule. The force that keeps the atoms together, is a complex force, modeled by approximations in quantum physics and chemistry, but it can usually be modeled as an elastic force, coming from a spring of negligible relaxed length. The two main forces that appear in this system are:

- The electrostatic repulsion between the hydrogen nuclei, $F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$
- The "elastic" bond force, $F = kr$

In the above formulas, r is the length of the bond. Knowing that the equilibrium bond length in the H_2 molecule is approximately $r_0 = 0.74\text{Å}$:

D1. Find the stiffness k of the bond.

(0.50 points)

The bond length of molecules is not however constant - it changes slightly over time, oscillating around the equilibrium position - this is known as the harmonic oscillator model. We can show that the period of these oscillations is given by:

$$T = 2\pi\sqrt{\frac{\mu}{k}}$$

Where μ is the reduced mass of the molecule, which is given by $\mu = \frac{m_1 m_2}{m_1+m_2}$ (with m_1 and m_2 being the masses of the two atoms in a molecule).

D2. If the mass of one hydrogen atom is $m = 1.67 \times 10^{-27}\text{kg}$, find the frequency of its oscillations.

(0.70 points)

Over the course of the oscillation, the bond length r increases (and then decreases) by approximately 17%.

D3. Find the maximum bond length r_{max} and the minimum r_{min}

(0.30 points)

D4. If the oscillation energy of the hydrogen molecule is lowered and the amplitude of the oscillations decreases from 17% to 5%, what will happen to the frequency?

- ☐ It will get lower
- ☐ It will stay the same
- ☐ It will get higher

(0.20 points)

Problem 3 – A Simple Experiment (4.00 points)

Using the Phyphox app, we can obtain and analyze raw data from almost all physical sensors inside our phones. One particularly interesting sensor is the accelerometer of the phone.

The following experiment was performed:

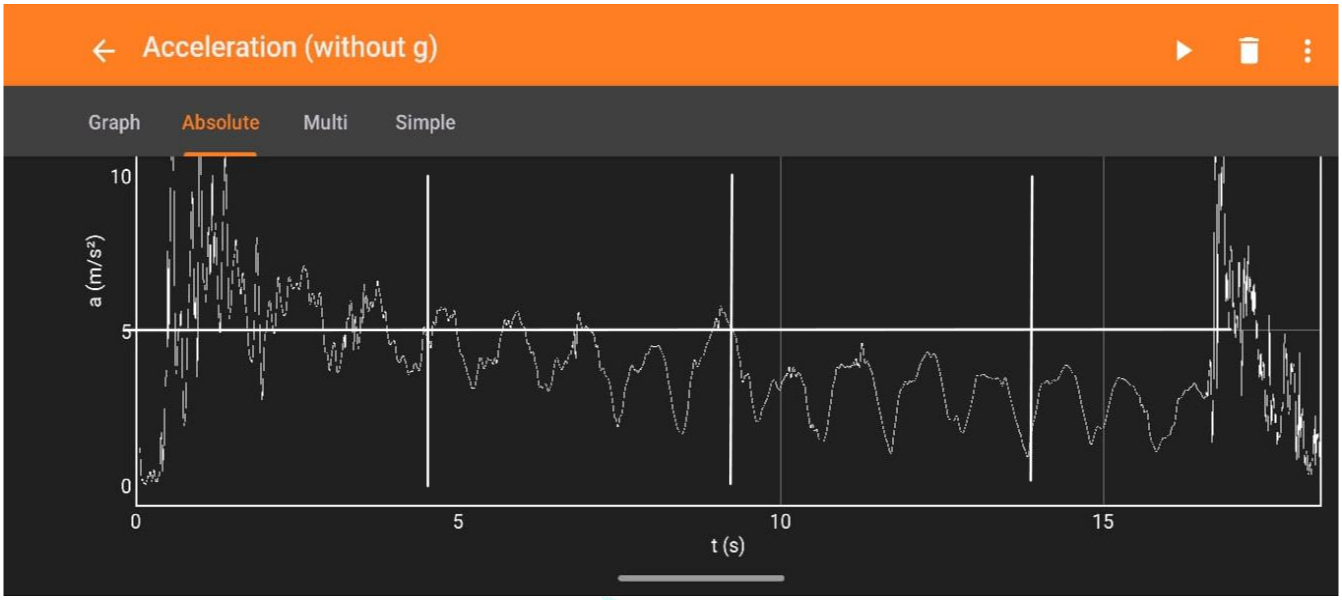
1. An almost inextensible rope was hanged from a fixed point
2. At the other end of the rope, a bag was put, in which the phone was put and secured to not fall out
3. The system was deviated from its vertical equilibrium position, essentially creating a gravitational pendulum
4. The accelerometer of the phone was started and the system was left to oscillate freely

In this problem, you might need to use the formula for the period of a gravitational pendulum:

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Where L is the length of the pendulum and g is the acceleration due to gravity.

The results are shown in the graph below (you will be provided with a larger version on the answer sheet):



Part A – Finding the Length of this Rope (2.20 points)

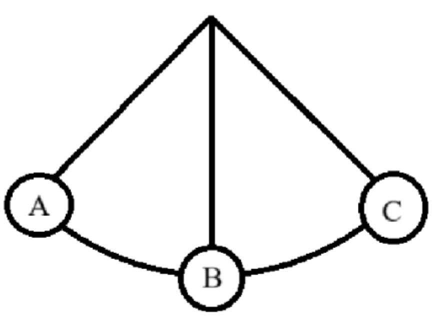
For this part, consider the rope to be perfectly inextensible.

A1. Note the approximate moment at which the phone was released and the moment at which it was caught again to check the obtained data.

(0.30 points)

A2. In which of the position (as shown in the picture below – A is the position from which the phone was released) is the acceleration maximum?

(0.25 points)



A3. A period is defined as the time the phone takes to go back to position A. How many times does the phone pass through B during an entire period?

(0.20 points)

A4. Using the data in the graph, estimate the period of the studied pendulum.

(0.75 points)

A5. Knowing the gravitational acceleration $g = 9.81 \frac{m}{s^2}$, find the length of the rope.

(0.75 points)

A6. Why does the average acceleration overall decrease in time?

- ☐ Due to inaccuracies in the sensor of the phone
- ☐ Due to air friction and similar dissipative forces
- ☐ That is the normal behavior of an ideal gravitational pendulum

(0.25 points)

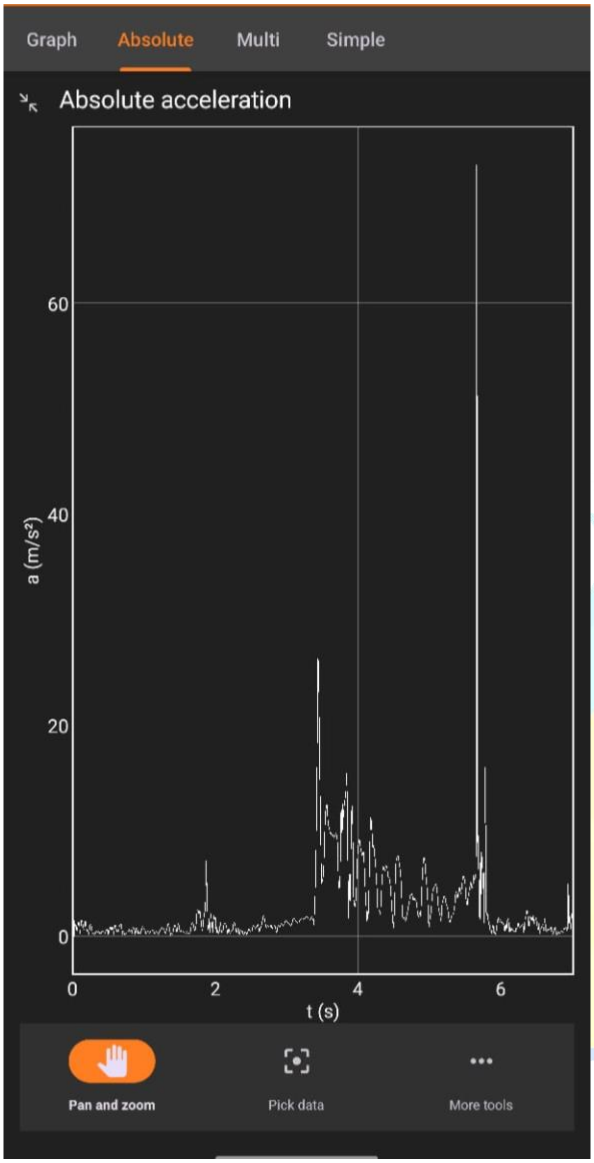
Part B – Elasticity of the Rope (1.80 points)

If you weren't able to solve part A5, use $L = 1m$. For this part, do not neglect elasticity.

The following experiment was done:

1. The phone was again put in the bag, in a setup identical to part A
2. Instead of an angular deviation, the bag with the phone is now pulled down by 10cm
3. After the system is pulled down, the accelerometer is started and, after some time, the system is left free, launching the phone upwards

The obtained data is in the following graph:



B1. Identify the moment at which the phone was launched upwards.

(0.15 points)

B2. Using the peak acceleration corresponding to the launch moment and knowing that the mass of the phone is $m = 197\text{g}$, find the stiffness K of the spring.

(0.75 points)

If you couldn't solve B2, use $K = 100\ \text{N/m}$.

At the moment when, during the gravitational oscillations (part A), the given acceleration is maximum, we can calculate the elongation ΔL of the rope.

B3. Find the relative elongation of the rope at that moment (the percent by which the rope gets longer)

(0.75 points)

B4. According to your result in part B3, is neglecting elasticity in part A reasonable?

(0.15 points)

Problem 4 – Meeting on the Vertical (4.00 points)

From the top of a tower of height H = 20m, stone A is dropped from rest. At the same moment, another stone B is thrown vertically upwards from the base of the tower with an initial speed $v_0 = 10\frac{m}{s}$. Air resistance is negligible.

Part A – The Meeting (2.00 points)

A1. Determine the time t after launch at which the stones will meet.

(0.75 points)

A2. At what height h (measured from the ground) will this happen?

(0.50 points)

A3. What condition must the initial speed v_0 satisfy for a collision to occur at all (i.e., before stone A hits the ground)?

(0.75 points)

Part B – The Collision (2.00 points)

For this part, consider a different set of initial conditions, which leads to the collision happening at height $h = 10m$. The velocities of the two stones are $v_A = 10\frac{m}{s}$, downwards and $v_B = 7.5\frac{m}{s}$, upwards.

B1. Assuming the collision is perfectly inelastic (plastic) and that $m_B = 2m_A$, find the velocity of the stones after the collision. Is it directed upwards or downwards?

(0.50 points)

B2. Assuming the collision is perfectly elastic and that $m_A = m_B$, find the maximum height that stone A reaches after it rebounds.

(1.00 points)

B3. In the conditions from B2, find the impulse (J or Δp) stone A exerts on stone B during the collision. Take $m_B = 1kg$.

(0.50 points)