Physics Mock Test No. 1

IJSO Theory Mock Test

Solutions

Problem 1 – Airborne Aircraft Carrier

Part A – Takeoff and Landing

A1. Find the minimum take-off speed of the aircraft (relative to Earth) at its maximum weight capacity.

Calculation:

Writing the Bernoulli equation for the air above and below the wings and neglecting the last term (wings are thin enough):

$$P_{above} + \frac{1}{2}\rho(v+5)^2 = P_{below} + \frac{1}{2}\rho(v-5)^2$$

(0.40 points)

The pressure difference which generates lift is:

$$\Delta P = P_{below} - P_{above} = \frac{1}{2}\rho((v+5)^2 - (v-5)^2) = \frac{1}{2}\cdot 1.2\cdot 4\cdot 5v = 12v$$

(0.30 points)

The area of a wing is $S = \frac{1}{2}\cdot 100\cdot 200 = 10^4 m^2$

(0.10 points)

The total lift force generated by both wings is $F_{lift} = 2\Delta PS = 2.4\cdot 10^5 v$

(0.20 points)

To get the minimum takeoff velocity, we set $F_{lift} = M_{total}g$, numerically:

$$2.4\cdot 10^5 v = mg = (5560 + 20\cdot 22)\cdot 1000\cdot g = 5.88\cdot 10^7 \Longrightarrow v = 245\,\frac{m}{s}$$

(0.30 points)

The velocity v is relative to the wind. To get the takeoff velocity, we calculate it as $u = v \pm v_{wind}$. To minimize the takeoff velocity, we suppose the plane goes opposite to the wind, so $u = v - v_{wind} = 240\,\frac{m}{s}$

(0.20 points)

A2. Find how long the runway should be in order to reach the minimum takeoff velocity calculated in A1.

Calculation:

The work done by the engines in the first 200m is:

$$W_1 = F_{avg} \cdot D = \frac{40\% + 100\%}{2} F_0 \cdot D = 0.7F_0D$$

(0.70 points)

After that, the aircraft moves for a distance x until it reaches the takeoff velocity required. The work done is $W_2 = F_0 x$

(0.30 points)

From the work energy theorem, $\frac{1}{2} M_{total} u^2 = W_1 + W_2$

(0.50 points)

Solving for x, we get $x = \frac{\frac{1}{2} M_{total} u^2}{F_0} - 0.7D = 19060\text{m}$

(0.30 points)

The total runway length is $L = D + x = 19260\text{m} = 19.26\text{km}$

(0.20 points)

A3. Calculate the new takeoff velocity needed if each thruster provides maximum power.

Calculation:

In this case, we can write $F_{lift} = M_{total} g - 150F_{thruster} = 1.6 \cdot 10^7 \text{N}$

(0.20 points)

Using the result from A1, $u = \frac{1.6 \cdot 10^7}{2.4 \cdot 10^5} - 5 = 61.67 \frac{\text{m}}{\text{s}}$

(0.30 points)

A4. Calculate the new runway length in the same conditions as A2.

Calculation:

We can just use the formula from A2, $L = \frac{\frac{1}{2} M_{total}\, u^2}{F_0} + 0.3D = 1327.6\text{m} = 1.33\text{km}$

A5. Find how long a runway should be in order for the craft to successfully land.

Calculation:

The total force which makes the aircraft stop is $F_{stop} = 1.2 \cdot 10^7 \text{N}$

(0.10 points)

From the work energy theorem, $\frac{1}{2} M_{total}\, u^2 = F_{stop}\, L_{min}$

(0.30 points)

Solving for L_{min} we get $L_{min} = \frac{\frac{1}{2} M_{total} u^2}{F_{stop}} = 950.7\ \text{m}$

(0.10 points)

Part B – Fuel source

B1. Find v.

Calculation:

The density of air at 11km altitude is $\rho = 0.34 \frac{\text{kg}}{\text{m}^3}$

(0.25 points)

Using the same reasoning as in A1, $\Delta P = 3.4v$, $F_{lift} = 2\Delta P S = 6.8 \cdot 10^4 v$

(0.30 points)

$$F_{lift} = M_{total} g \Longrightarrow v = \frac{5.89 \cdot 10^7}{6.8 \cdot 10^4} = 866 \frac{\text{m}}{\text{s}}$$

(0.20 points)

B2. Find the power exerted by one engine if the airplane has 4 engines.

Calculation:

The force exerted by the engines is $F = \frac{1}{2}\rho C_D A v^2 = 7.65 \cdot 10^7 N$

(0.20 points)

The power is $P = F \cdot v = 6.625 \cdot 10^{10} W$

(0.35 points)

The power exerted by one engine is $\frac{P}{4} = 1.66 \cdot 10^{10} W$

(0.20 points)

B3. Find the amount of fuel needed to keep this craft moving at a fixed altitude for 40 days.

Calculation:

$$M = \frac{6.625 \cdot 10^{10} W \cdot 40 days \cdot 24 \frac{h}{day} \cdot 3600 \frac{s}{h}}{43.2 \cdot 10^6 \frac{J}{kg}} = 5.3 \cdot 10^9 kg$$

B4. Find the mass of the uranium sample needed to keep this craft flying for 40 days.

Calculation:

$$M = \frac{6.625 \cdot 10^{10}\, W \ \cdot \ 40 days \ \cdot \ 24 \frac{h}{day} \ \cdot \ 3600 \frac{s}{h}}{0.03 \frac{g\, U-235}{g\ sample} \ \cdot \ 8.21 \cdot 10^{10} \frac{J}{g\ sample}} = 9.30 \cdot 10^4 kg$$

B5. Which fuel source is more suitable?

Uranium is more suitable as it has a higher energy output

Part C – Density of the Airplane

C1. What is the mass percent composition of an alloy of aluminum and titanium having a density of 2.90 g / cm^3?

Calculation:

Let w be the mass percent of aluminum. The volume of the alloy is:

$$V = 0.95\left(\frac{wM}{\rho_{Al}} + \frac{(1-w)\,M}{\rho_{Ti}}\right) = M \cdot 0.95\left(\frac{w}{\rho_{Al}} + \frac{1-w}{\rho_{Ti}}\right)$$

(0.30 points)

The density of the alloy is $\rho = \frac{M}{V} = \frac{M}{M \cdot 0.95\left(\frac{w}{\rho_{Al}}+\frac{1-w}{\rho_{Ti}}\right)} = \frac{\rho_{Al}\rho_{Ti}}{0.95(w\rho_{Ti}+(1-w)\rho_{Al})}$

(0.40 points)

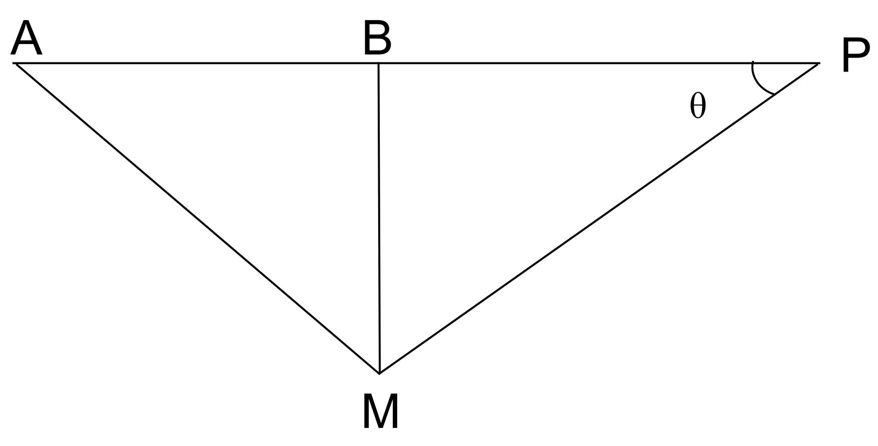
Solving for w, we get $w = \frac{\rho_{Al}\rho_{Ti}-0.95\rho\rho_{Al}}{0.95\rho(\rho_{Ti}-\rho_{Al})} = 0.95 = 95\%$, so the composition is 95% Al + 5% Ti

(0.30 points)

Part D – Altitude

D1. Find the speed of the plane.

Calculation:



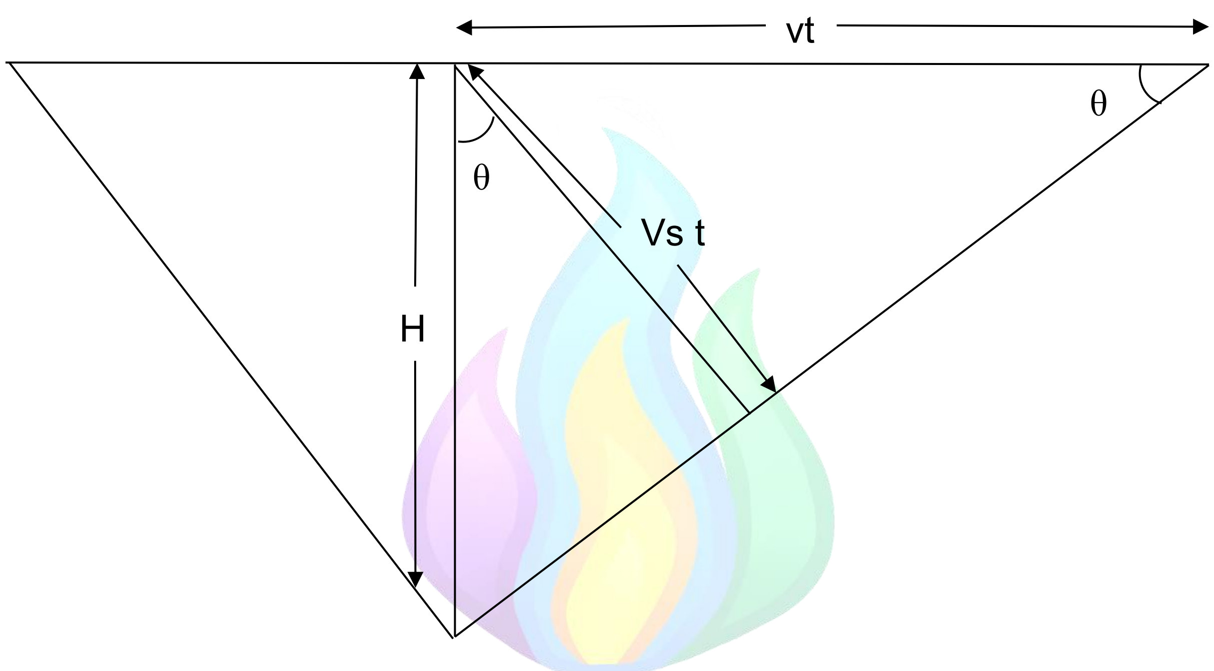
A - origin of the first sound the man hears.

B - place directly over the man

M - place where the man in standing

P - position of the airplane

$\angle AMP = 90^\circ$ because MP is the tangent to the circle with radius AM and a centre M.



The distance marked as $v_s t$ is found by applying $\sin\theta = \frac{v_s}{v}$.

(0.50 points)

$$\sin\theta = \frac{\sqrt{H^2 - {v_s}^2 t^2}}{H} = \frac{v_s}{v}$$

(0.50 points)

$$\frac{H^2 - {v_s}^2 t^2}{H^2} = \frac{{v_s}^2}{v^2}$$

$$H^2 v^2 - {v_s}^2 v^2 t^2 = {v_s}^2 H^2$$

$$v^2 = \frac{{v_s}^2 H^2}{H^2 - {v_s}^2 t^2}$$

(0.50 points)

$$v^2 = \frac{(330 \cdot 4000)^2}{4000^2 - 3300^2} = 340978$$

$$v = 583.93 \text{ m/s}$$

(0.25 points)

Problem 2 – An experimental analysis of star luminosity

A. Find a formula for the luminosity per unit area of the star M, using the conservation of energy and assuming a spherically symmetric distribution of the energy.

Calculation:

The flux measured at Earth, f, equals the star's total luminosity L spread over a sphere of radius d:

$$f = \frac{L}{4\pi d^2} \rightarrow L = 4\pi d^2 f$$

(0.75 points)

The luminosity per unit area on the star's surface is

$$M = \frac{L}{4\pi R^2} = \frac{4\pi d^2 f}{4\pi R^2} = f\frac{d^2}{R^2}$$

(0.75 points)

B. Fill in the following table:

Star	δ (rad)	d (m)	R (m)	M ($\frac{W}{m^2}$)
Arcturus	$1.018 \cdot 10^{-7}$	$3.491 \cdot 10^{17}$	$1.777 \cdot 10^{10}$	$1.895 \cdot 10^{7}$
Vega	$1.590 \cdot 10^{-8}$	$2.381 \cdot 10^{17}$	$1.893 \cdot 10^{9}$	$4.889 \cdot 10^{8}$
Sirius A	$2.928 \cdot 10^{-8}$	$8.184 \cdot 10^{16}$	$1.198 \cdot 10^{9}$	$5.320 \cdot 10^{8}$
Dubhe	$1.454 \cdot 10^{-8}$	$1.169 \cdot 10^{18}$	$8.499 \cdot 10^{9}$	$2.857 \cdot 10^{7}$
Procyon	$2.618 \cdot 10^{-8}$	$1.088 \cdot 10^{17}$	$1.424 \cdot 10^{9}$	$1.057 \cdot 10^{8}$

Extra space:

$$1\ \text{mas} = \frac{1}{1000} \times \frac{1}{3600} \times \frac{\pi}{180} = 4.848 \times 10^{-9}\ \text{rad}$$
$$1\ \text{pc} = 3.1 \times 10^{16}\ \text{m}$$
$$R = \frac{\delta d}{2}$$
$$M = f\frac{d^2}{R^2}$$

Substituting values for Arcturus,

$$\delta = 21.0\ \text{mas} = 21.0 \times 4.848\ \cdot 10^{-9} = 1.018 \cdot 10^{-7}\ \text{rad}$$

$$d = 11.26\ \text{pc} = 11.26 \times 3.1 \cdot 10^{16} = 3.491 \cdot 10^{17}\ \text{m}$$

$$R = \frac{\delta d}{2} = \frac{1.018\ \cdot\ 10^{-7}\ \times\ 3.491\ \cdot\ 10^{17}}{2} = 1.777 \cdot 10^{10}\ \text{m}$$

$$M = f\frac{d^2}{R^2} = 4.91 \cdot 10^{-8} \times \frac{(3.491\ \cdot\ 10^{17})^2}{(1.777\ \cdot\ 10^{10})^2} = 1.895 \cdot 10^{7}\ \text{Wm}^{-2}$$

Substituting values for Vega,

$$\delta = 3.28\ \text{mas}\ = 3.28 \times 4.848 \cdot 10^{-9} = 1.590 \cdot 10^{-8}\ \text{rad}$$

$$d = 7.68\ \text{pc} = 7.68\ \times 3.1 \cdot 10^{16} = 2.381 \cdot 10^{17}\ \text{m}$$

$$R = \frac{\delta d}{2} = \frac{1.590\ \cdot\ 10^{-8}\ \times\ 2.381 \cdot 10^{17}}{2} = 1.893 \cdot 10^{9}\ \text{m}$$

$$M = f\frac{d^2}{R^2} = 3.09 \cdot 10^{-8} \times \frac{(2.381\ \cdot\ 10^{17})^2}{(1.893\ \cdot\ 10^{9})^2} = 4.889 \cdot 10^{8}\ \text{Wm}^{-2}$$

Substituting values for Sirius A,

$$\delta = 6.04 \text{ mas } = 6.04 \times 4.848 \cdot 10^{-9} = 2.928 \cdot 10^{-8} \text{ rad}$$

$$d = 2.64 \text{ pc} = 2.64 \times 3.1 \cdot 10^{16} = 8.184 \cdot 10^{16} \text{ m}$$

$$R = \frac{\delta d}{2} = \frac{2.928 \cdot 10^{-8} \times 8.184 \cdot 10^{16}}{2} = 1.198 \cdot 10^{9} \text{ m}$$

$$M = f\frac{d^2}{R^2} = 1.14 \cdot 10^{-7} \times \frac{(8.184 \cdot 10^{16})^2}{(1.198 \cdot 10^{9})^2} = 5.320 \cdot 10^{8} \text{ Wm}^{-2}$$

Substituting values for Dubhe,

$$\delta = 3.00 \text{ mas } = 3.00 \times 4.848 \cdot 10^{-9} = 1.454 \cdot 10^{-8} \text{ rad}$$

$$d = 37.7 \text{ pc} = 37.7 \times 3.1 \cdot 10^{16} = 1.169 \cdot 10^{18} \text{ m}$$

$$R = \frac{\delta d}{2} = \frac{1.454 \cdot 10^{-8} \times 1.169 \cdot 10^{18}}{2} = 8.499 \cdot 10^{9} \text{ m}$$

$$M = f\frac{d^2}{R^2} = 1.51 \cdot 10^{-9} \times \frac{(1.169 \cdot 10^{18})^2}{(8.499 \cdot 10^{9})^2} = 2.857 \cdot 10^{7} \text{ Wm}^{-2}$$

Substituting values for Procyon,

$$\delta = 5.40 \text{ mas } = 5.40 \times 4.848 \cdot 10^{-9} = 2.618 \cdot 10^{-8} \text{ rad}$$

$$d = 3.51 \text{ pc} = 3.51 \times 3.1 \cdot 10^{16} = 1.088 \cdot 10^{17} \text{ m}$$

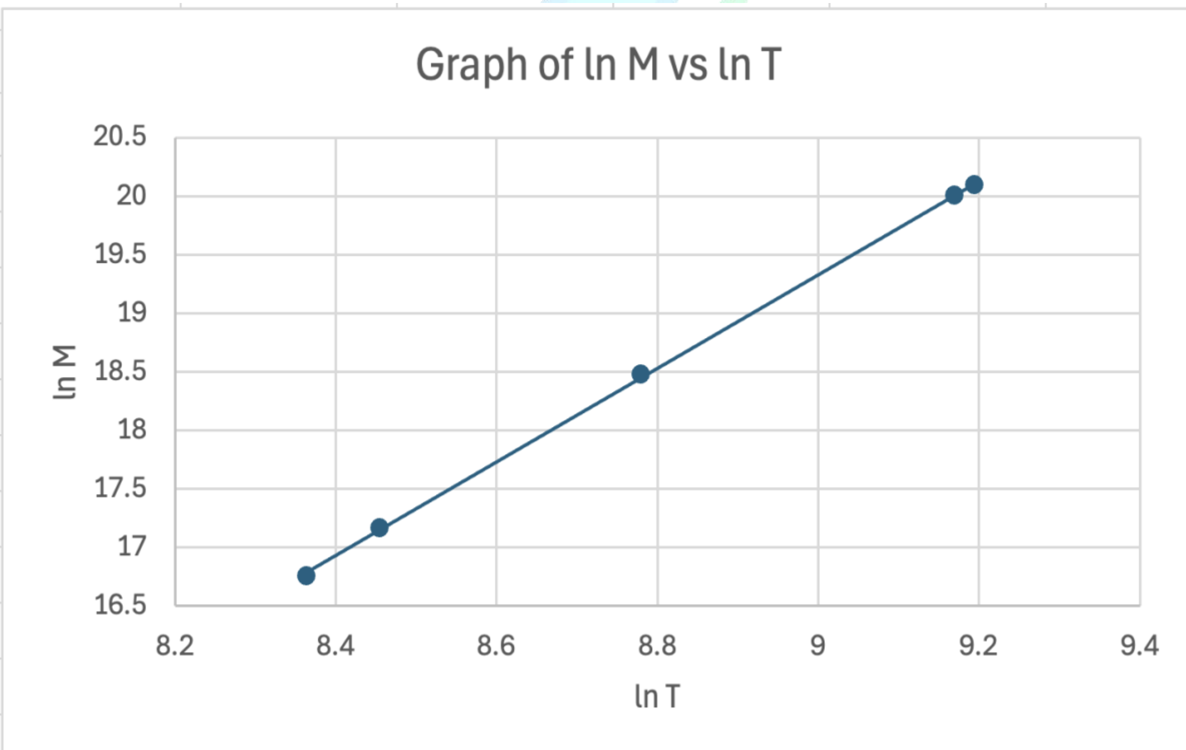
$$R = \frac{\delta d}{2} = \frac{2.618 \cdot 10^{-8} \times 1.088 \cdot 10^{17}}{2} = 1.424 \cdot 10^{9} \text{ m}$$

$$M = f\frac{d^2}{R^2} = 1.81 \cdot 10^{-8} \times \frac{(1.088 \cdot 10^{17})^2}{(1.424 \cdot 10^{9})^2} = 1.057 \cdot 10^{8} \text{ Wm}^{-2}$$

The data is marked as follows: 0.15p x 5 angular diameters, 0.10p x 5 distances, 0.20p x 5 radii, 0.25p x 5 fluxes. If the result from A is wrong, but the values of M are calculated correctly (according to the wrong formula from A), full marks are awarded for the M values.

C. On a sheet of graph paper, graph ln M as a function of ln T.

Star	ln M	ln T
Arcturus	16.757	8.364
Vega	20.008	9.170
Sirius A	20.092	9.195
Dubhe	17.168	8.455
Procyon	18.476	8.780



Marking scheme: 0.20p per correctly represented point, 0.20p for choosing the axis properly (lnM - y and lnT - x), 0.40p for choosing a scale and numbering the axis accordingly, 0.40p for finding the best fit line. Award 1.00p if data points cover less than ⅔ rd of the graph's surface

D. Using the graph, find the value of n.

Calculation:

Picking two points on the best-fit line = (8.4, 16.9) , (9.0, 19.3)

(0.75p, only 0.30p if the points are chosen from the table not from the line)

Gradient = $\frac{\Delta y}{\Delta x} = \frac{19.3 - 16.9}{9.0 - 8.4} = \frac{2.4}{0.6} = 4.0$

(0.75 points)

n = Gradient = 4.0

E. Using the found value of n, calculate the value of σ for each of the stars. Calculate the mean value.

Solution 1

$\ln \sigma = \ln M - n \ln T$

Star	ln M	ln T	ln M - n ln T
Arcturus	16.757	8.364	-16.699
Vega	20.008	9.170	-16.672
Sirius A	20.092	9.195	-16.688
Dubhe	17.168	8.455	-16.652
Procyon	18.476	8.780	-16.644

(0.25x5 = 1.25 points)

The average $= \frac{(-16.999) + (-16.672) + (-16.688) + (-16.652) + (-16.644)}{5} = -16.671$

$\sigma = e^{\ln \sigma} = e^{-16.671} = 5.753 \cdot 10^{-8}$

(0.25 points)

1.25 points shall be awarded if the student took the average of σ instead of ln σ

Solution 2

$\sigma = \frac{M}{T^4}$

Star	M	T	$\frac{M}{T^4}$
Arcturus	$1.895 \cdot 10^7$	4290	$5.595 \cdot 10^{-8}$
Vega	$4.889 \cdot 10^8$	9600	$5.756 \cdot 10^{-8}$
Sirius A	$5.320 \cdot 10^8$	9845	$5.663 \cdot 10^{-8}$
Dubhe	$2.857 \cdot 10^7$	4700	$5.855 \cdot 10^{-8}$
Procyon	$1.057 \cdot 10^8$	6500	$5.921 \cdot 10^{-8}$

(5x0.25 = 1.25 points)

The average $= \frac{(5.595 + 5.756 + 5.663 + 5.885 + 5.921) \cdot 10^{-8}}{5} = 5.758 \cdot 10^{-8}$

(0.25 points)

Problem 3 – Climate Physics

Part A – Modelling cloud formation

A1. What phenomenon takes place when clouds form?

The phenomenon that takes place when clouds form is condensation. Clouds form when water vapor in the air cools and changes into tiny liquid water droplets or ice crystals

A2. Fill the following table:

Calculation:

$$T = T_0 - h\Gamma$$

h/km	T/K
0.5	286.75
1.0	283.50
1.5	280.25
2.0	277.00
2.5	273.75
3.0	270.50
3.5	267.25

(0.10x7 = 0.70 points)

A3. Fill the following table with values of the partial pressure of water, knowing that the partial pressure of a gas in a mixture is directly proportional to its mole fraction:

h/km	P_{H_2O} / Pa
0.0	1500
0.5	1252
1.0	1045
1.5	871.9
2.0	728.1
2.5	607.7
3.0	507.3
3.5	423.5

Extra space:

$$e^{-\frac{Mg}{RT_0}h} = e^{-\frac{0.0289 \times 9.80}{8.314 \times 290}h} = e^{-1.1747 \cdot 10^{-4}h}$$

For 0.0 km,

$P(0) = P_0 = 1.000 \cdot 10^5$ Pa

$x(0) = x_0 = 0.01500$

$P_{H_2O}(0) = P \times x = 1.000 \cdot 10^5 \times 0.01500 = 1500$

For 0.5 km,

$P\,(500) = P_0 e^{-1.1747 \cdot 10^{-4} h} = P_0 e^{-1.1747 \cdot 10^{-4} \times 500} = 9.430 \cdot 10^4 \text{ Pa}$

$x\,(500) = x_0 e^{-\frac{500}{4.1 \times 10^3}} = 0.01328$

$P_{H_2O}\,(500) = P \times x = 9.430 \cdot 10^4 \times 0.01328 = 1252$

For 1.0 km,

$P\,(1000) = P_0 e^{-1.1747 \cdot 10^{-4} h} = P_0 e^{-1.1747 \cdot 10^{-4} \times 1000} = 8.892 \cdot 10^4 \text{ Pa}$

$x\,(1000) = x_0 e^{-\frac{1000}{4.1 \times 10^3}} = 0.01175$

$P_{H_2O}\,(1000) = P \times x = 8.892 \cdot 10^4 \times 0.01175 = 1045$

For 1.5 km,

$P\,(1500) = P_0 e^{-1.1747 \cdot 10^{-4} h} = P_0 e^{-1.1747 \cdot 10^{-4} \times 1500} = 8.384 \cdot 10^4 \text{ Pa}$

$x\,(1500) = x_0 e^{-\frac{1500}{4.1 \times 10^3}} = 0.01040$

$P_{H_2O}\,(1500) = P \times x = 8.384 \cdot 10^4 \times 0.01040 = 871.9$

For 2.0 km,

$P\,(2000) = P_0 e^{-1.1747 \cdot 10^{-4} h} = P_0 e^{-1.1747 \cdot 10^{-4} \times 2000} = 7.906 \cdot 10^4 \text{ Pa}$

$x\,(2000) = x_0 e^{-\frac{2000}{4.1 \times 10^3}} = 9.210 \cdot 10^{-3}$

$P_{H_2O}\,(2000) = P \times x = 7.906 \cdot 10^4 \times 9.210 \cdot 10^{-3} = 728.1$

For 2.5 km,

$P\,(2500) = P_0 e^{-1.1747\cdot 10^{-4} h} = P_0 e^{-1.1747\cdot 10^{-4}\times 2500} = 7.455\cdot 10^4$ Pa

$x\,(2500) = x_0 e^{-\frac{2500}{4.1\times 10^3}} = 8.152\cdot 10^{-3}$

$P_{H_2O}\,(2500) = P\times x = 7.455\cdot 10^4 \times 8.152\cdot 10^{-3} = 607.7$

For 3.0 km,

$P\,(3000) = P_0 e^{-1.1747\cdot 10^{-4} h} = P_0 e^{-1.1747\cdot 10^{-4}\times 3000} = 7.030\cdot 10^4$ Pa

$x\,(3000) = x_0 e^{-\frac{3000}{4.1\times 10^3}} = 7.216\cdot 10^{-3}$

$P_{H_2O}\,(3000) = P\times x = 7.030\cdot 10^4 \times 7.216\cdot 10^{-3} = 507.3$

For 3.5 km,

$P\,(3500) = P_0 e^{-1.1747\cdot 10^{-4} h} = P_0 e^{-1.1747\cdot 10^{-4}\times 3500} = 6.629\cdot 10^4$ Pa

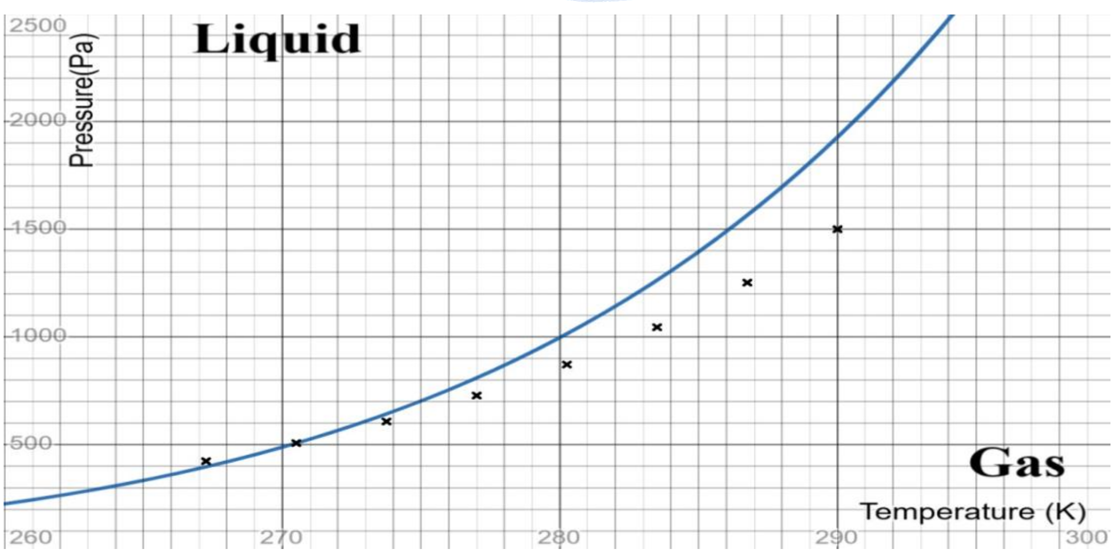
$x\,(3500) = x_0 e^{-\frac{3500}{4.1\times 10^3}} = 6.388\cdot 10^{-3}$

$P_{H_2O}\,(3500) = P\times x = 6.629\cdot 10^4 \times 6.388\cdot 10^{-3} = 423.5$

0.16 points for correct formula and 0.18 points per value. If all values are correct but the formula isn't written, full marks are awarded.

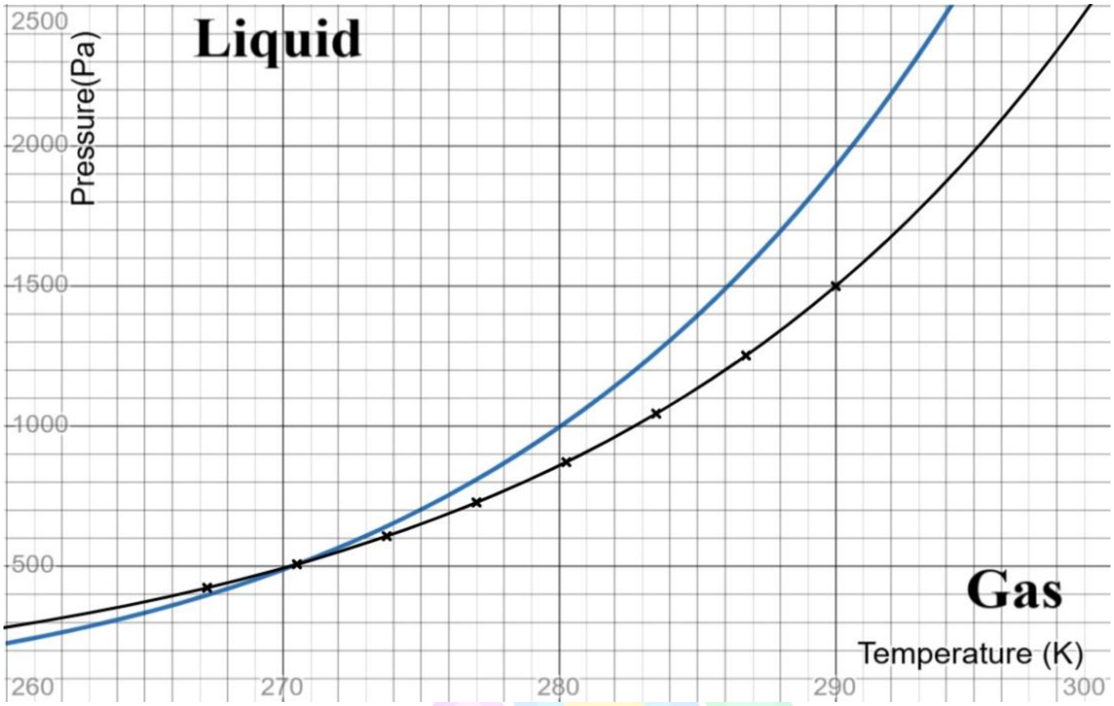
A4. On the phase diagram, plot each of the found (T, P_{H2O}) points. It might be useful to fill the following table (no points will be deducted for not filling it out):

h/km	T/K	P_{H_2O}/Pa
0.0	290.00	1500
0.5	286.75	1252
1.0	283.50	1045
1.5	280.25	871.9
2.0	277.00	728.1
2.5	273.75	607.7
3.0	270.50	507.3
3.5	267.25	423.5



(0.15x8 = 1.20 points)

A5. On the phase diagram, trace a curve going through all the points drawn in part A4.



(0.50 points, 0.20 points if a line is drawn)

A6. What is the temperature at the height where clouds are expected to form? Using that, at what height do you expect clouds to form?

Calculation:

Clouds form when the temperature is around 270.5 K.

0.30 points for $270<T<271$

0.20 points for $269<T<271$

0.10 points for $267<T<273$

$$T = T_0 - h\Gamma$$

$$h = \frac{T_0 - T}{\Gamma} = \frac{290 - 270.5}{6.5} = 3\ \mathrm{km} = 3000\ \mathrm{m}$$

Part B – Cloud electrification

B1. Consider two neutral clouds, after a frictional contact between them, they will end up being charged, what type of charge will each cloud have?

Charge is conserved throughout the process. Therefore, one cloud must be positive while the other is negative.

B2. Calculate the new charge of cloud 1 and the new charge of cloud 2.

Calculation:

Total charge = $200\ \mathrm{m}^3 \times \frac{10}{100} \times 5\ \mathrm{C/m}^3 = 100\ \mathrm{C}$

(0.20 points)

This charge will be equally distributed among the two clouds. So, each cloud will have 50 C each.

(0.40 points)

B3. The electric field due to cloud 1 on the ground, in a point right below it.

Calculation:

$$\text{Field} = \frac{\mathrm{kQ}}{\mathrm{r}^2} = \frac{9 \cdot 10^9 \times 50}{(5000)^2} = 18000\ \mathrm{N/C}$$

(0.60 points formula + 0.25 points value)

B4. The electric potential due to cloud 1 on the same point.

Calculation:

Potential = $\frac{\mathrm{kQ}}{\mathrm{r}} = \frac{9 \cdot 10^{9} \times 50}{(5000)} = 9 \cdot 10^{7}$ V

(0.60 points formula + 0.25 points value)

B5. Despite the potential difference between a cloud and the ground, we cannot assure that a lightning strike will actually occur, what is the best explanation for that?

- As shown in B4, the potential difference is quite large, so the first option is wrong.
- Air resistivity is very high ($10^{13}\Omega \cdot \mathrm{m}$ order of magnitude). High potential differences are "compensated" by the high resistivity and high enough currents (that would lead to ionization and lightning strikes) hardly form.
- The charge of objects on the ground doesn't have anything to do with electric discharge, as the charge that is transferred through lightning comes from the cloud. Charge on the ground would only repel the charge from the cloud, preventing the lightning from forming.
- The fourth option is true, but it does not explain why lightning doesn't form, it's the opposite - it explains how clouds quickly go through discharge - the discharge is known as lightning.

Part C – A different model for thunderstorms

C1. Assuming that air has the same electrical permittivity as vacuum, find the capacitance of the system formed by the cloud and the ground under it.

Calculation:

R = 5 km; H = 5 km.

$$C = \varepsilon \frac{A}{d}$$

(0.20 points)

$$A = \pi R^2$$

(0.10 points)

$$C = \varepsilon_0 \frac{\pi R^2}{d}$$

After substituting we get C = 0.14μF

(0.20 points)

C2. If the cloud is charged with Q = +100C, find the potential difference between the ground and the cloud.

Calculation:

$$Q = CV \rightarrow V = \frac{Q}{C} = \frac{100\ \mathrm{C}}{0.14\ \mu\mathrm{F}} = 714.29\ \mathrm{V}$$

C3. What value will the new potential difference between the ground and the cloud have?

Calculation:

$$\Delta Q = \frac{1}{2} \times Qt = \frac{1}{2} \times 2 \cdot 10^{6} \times 50 \cdot 10^{-6} = 50\ \mathrm{C}$$

$$Q_{new} = 100\ \mathrm{C} - 50\ \mathrm{C} = 50\ \mathrm{C}$$

$$V_{new} = \frac{kQ}{r} = \frac{9 \cdot 10^{9} \times 50}{5000} = 9 \cdot 10^{7}\ \mathrm{V}$$