Physics Mock Test No. 1

IJSO Theory mock test

This is an IJSO Physics mock test, designed to mimic the style, depth, and difficulty of chemistry questions found in the IJSO. Its aim is to help students strengthen their understanding of the physics concepts behind the IJSO and similar competitions.

The questions in this paper were made by the following past IJSO participants (in alphabetical order):

- Alex Jicu (Romania)
- Jailson Godeiro (Brazil)
- Thenura Wickramaratna (Sri Lanka) – Physics Mock Test no. 1 Coordinator

No.	Problem	Author	Marks
1	Airborne Aircraft Carrier	Thenura Wickramaratna	10.00
2	An Experimental Analysis of Star Luminosity	Alex Jicu	10.00
3	Climate Physics	Alex Jicu Jailson Godeiro	10.00

In solving the questions, you might need to use the following constants:

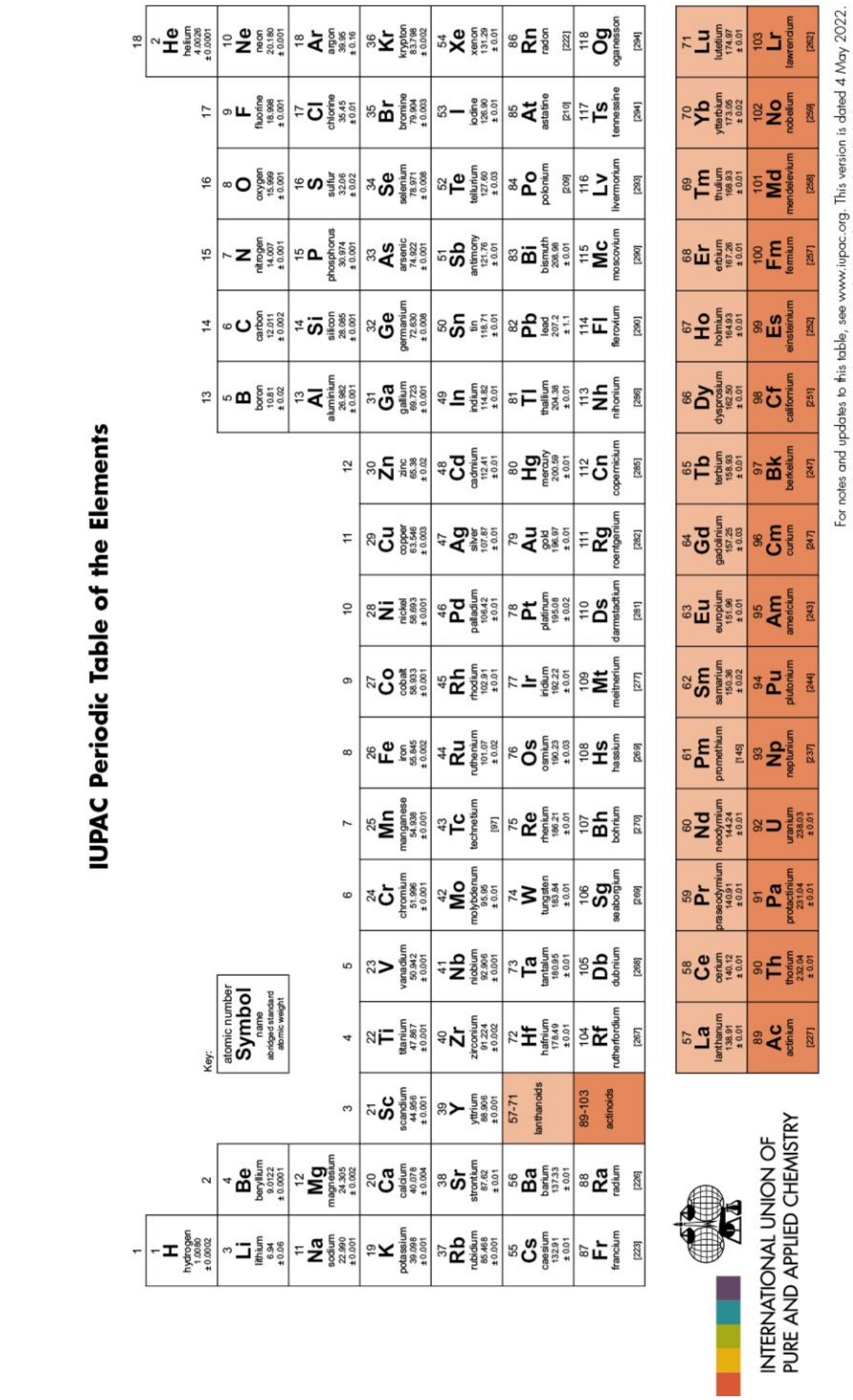
Constant	Notation	Value
Acceleration due to gravity	g	$9.8\ \mathrm{ms^{-2}}$
Gravitational constant	G	$6.67 \cdot 10^{-11}\ \mathrm{m^3/\,kg \cdot s^2}$
Planck's constant	h	$6.62 \cdot 10^{-34}\ \mathrm{J \cdot s}$
Elementary charge	e	$1.6 \cdot 10^{-19}\ \mathrm{C}$
Speed of light in vacuum	c	$3 \cdot 10^{8}\ \mathrm{ms^{-1}}$
Density of water	ρ	$1000\ \mathrm{kg\ m^{-3}}$
Stefan-Boltzmann constant	σ	$5.67 \cdot 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m^2K^4}}$
Universal gas constant	R	$8.314\ \mathrm{J\ mol^{-1}\ K^{-1}}$ $0.0821\ \mathrm{atm\ L\ mol^{-1}\ K^{-1}}$
Avogadro's number	N_A	$6.022 \cdot 10^{23}\ \mathrm{mol^{-1}}$
Faraday's constant	F	$96\,500\ \mathrm{C/mol}$
Pi	π	3.14
Electrical permittivity of free space	ε_0	$8.85 \cdot 10^{-12}\ \mathrm{F \cdot m^{-1}}$
Magnetic permeability of free space	μ_0	$4\pi \cdot 10^{-7}\ \mathrm{H/m}$
Mass of Earth		$5.97 \cdot 10^{24}\ \mathrm{kg}$
Mass of Moon		$7.35 \cdot 10^{22}\ \mathrm{kg}$
Mass of Sun		$1.99 \cdot 10^{30}\ \mathrm{kg}$
Radius of Earth		$6.4 \cdot 10^{6}\ \mathrm{km}$
Radius of Moon		$1.7 \cdot 10^{6}\ \mathrm{km}$
Radius of Sun		$6.96 \cdot 10^{8}\ \mathrm{km}$
Specific heat capacity of water	c_w	$4200\ \mathrm{J/kg \cdot {}^\circ C}$
Average molar mass of air	M	$28.9\ \mathrm{g/mol}$

If any other value is provided in the problem, use the value provided, not the one in the table. You can also use the following conversion formulas:

$T\ (\mathrm{K}) = t\ ({}^\circ\mathrm{C}) + 273$	$t\ ({}^\circ\mathrm{F}) = \frac{9}{5} t\ ({}^\circ\mathrm{C}) + 32$
$1\mathrm{bar} = 1\mathrm{atm} = 101\,000\mathrm{Pa} = 760\mathrm{mmHg}$	$1\mathrm{u} = 1\mathrm{Da} = 1.66 \cdot 10^{-27}\mathrm{kg}$
$1\mathrm{L} = 10^{-3}\ \mathrm{m^3}$	$1\ \mathrm{day} = 24\mathrm{h}$

If needed, you can use the periodic table given bellow:

(Use atomic masses rounded to two decimal places.)



Problem 1 – Airborne Aircraft Carrier - 10.00 points

In the 1960s, Lockheed Martin proposed a nuclear-powered airborne aircraft carrier named the CL-1201. Despite its ambitious design, the CL-1201 never advanced beyond the study phase, primarily due to the high costs and complexities associated with building such a large and nuclear-powered aircraft. In this problem, we will analyze the physics behind an aircraft like the CL-1201.

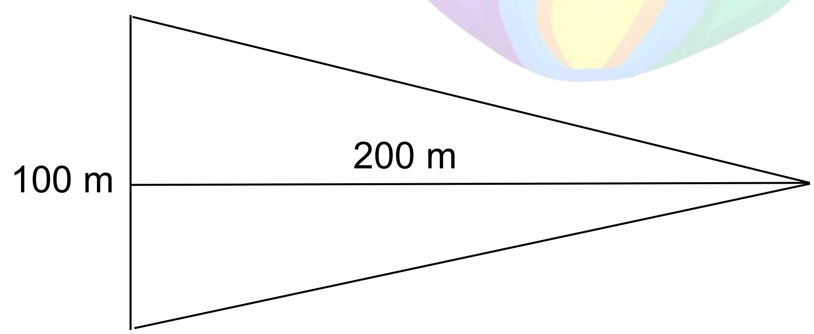
One of the most important equations when describing aircrafts and the way they maintain themselves in the air is Bernoulli's principle. It states that incompressible fluids (air can be considered incompressible throughout this problem) obey the equation:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

where P is the fluid's pressure, ρ is its density, v is its flow velocity, h is the altitude relative to a reference point, and g is the acceleration due to gravity.

Part A – Takeoff and Landing - 5.00 points

The carrier weighs 5560 tons without any aircraft inside, and an aircraft weighs 20 tons. At its maximum capacity, the carrier can accommodate 22 airplanes. The dimensions of the wing of the aircraft are given below.



The velocity of the natural wind with respect to the Earth is 5 m/s. Let us denote v as the relative speed of the aircraft with respect to the wind. The speed of the wind with respect to the Earth above and below the wing is $v + 5$ and $v - 5$, due to the aircraft's aerodynamic engineering. You can consider that only the aircraft's wings generate considerable lift. Assume the density of air to be $1.2\ \mathrm{kg/m^3}$.

A1. Find the minimum take-off speed of the aircraft (relative to Earth) at its maximum weight capacity.

(1.50 points)

The aircraft engines can exert a maximum force of 9×10^6 N during takeoff. Forces due to friction and air resistance are negligible. The aircraft's force output immediately before accelerating is 40%, and this gradually increases to 100% after travelling 200m.

A2. Find how long the runway should be in order to reach the minimum takeoff velocity calculated in A1.

(2.00 points)

The new aircraft model will have 150 lift-off thrusters, each producing a downward force of 272000 N on the surrounding air. By Newton's third law, the air exerts an equal upward reaction force on the aircraft, aiding its takeoff. Keep in mind the lift off thrusters are only used during takeoff.

A3. Calculate the new takeoff velocity needed if each thruster provides maximum power.

(0.50 points)

A4. Calculate the new runway length in the same conditions as A2.

(0.50 points)

During landing, the touchdown speed of the craft is equal to the takeoff velocity found in A3. The braking force due to the aircraft's braking mechanisms is found to be 4.0×10^6 N. Frictional force between the tires and the road during braking is found to be constant and equal to 8.0×10^6 N.

A5. Find how long a runway should be in order for the craft to successfully land.

(0.50 points)

Part B – Fuel Source - 2.25 points

The craft is flying at 11 km. The equation for density of air at an altitude h from the sea level is found to be $\rho(h) = \rho_0 \cdot e^{-\frac{Mgh}{RT}}$ where ρ_0 is the air density at sea level, M is the molar mass of air, g is the acceleration due to gravity, R is the universal gas constant, and T is the absolute temperature.

Air speed below and above the wing is $v - 5$ and $v + 5$ where v is the speed of the craft. Assume the wind speed relative to the Earth is zero.
B1. Find v.

(0.75 points)

The carrier travels with a uniform velocity equal to v found in part B1. There is a constant air resistant force $F = -\frac{1}{2}\rho C_D A v^2$ is acting against the carrier. In this equation, ρ = density of air, C_D = a coefficient (for aeroplanes, this coefficient can be assumed to be 0.02), A = area (for the CL-1201, this can be assumed to be $30000\ m^2$) and v = velocity of the aeroplane.

B2. Find the power exerted by one engine if the airplane has 4 engines.

(0.75 points)

B3. 1 kilogram of airplane fuel gives off 43.2 MJ of energy. Find the amount of fuel needed to keep this craft moving at a fixed altitude for 40 days.

(0.25 points)

B4. A uranium sample contains 3% U-235. U-235 is fissile and undergoes nuclear fission. 1 gram of pure U-235 fission emits 8.21×10^{10} J of energy. Find the mass of the uranium sample needed to keep this craft flying for 40 days.

(0.25 points)

B5. Which fuel source is more suitable?

- ☐ Airplane fuel
- ☐ Uranium sample

(0.25 points)

Part C – Density of the Airplane - 1.00 points

Generally, planes are made out of light but resistant alloys, having a density of around 2.90 g / cm^3. The main components of the alloys used in airplane making are aluminum (density in pure form 2.70 g / cm^3) and titanium (density in pure form 4.51 g / cm^3). Because when alloys are formed, one metal's atoms occupy free holes created in the lattice of the other metal, the total volume isn't exactly conserved. In this case, you may assume a 5% volume contraction.

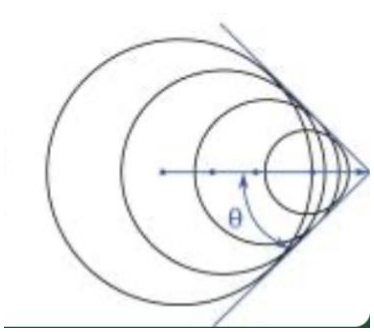
C1. What is the mass percent composition of an alloy of aluminum and titanium having a density of 2.90 g / cm^3?

(1.00 points)

Part D – Altitude - 1.75 points

A crew member (at ground) notices another craft flying at constant height H = 4 km with a constant supersonic velocity. The member hears the plane at t = 10 s after the moment when the plane was directly above him.

A supersonic craft generates a sonic boom. The angle of the sonic boom, θ (measured between the direction of motion of the craft and the cone surface), is such that $\sin\,\theta = \frac{v_s}{v}$ where v_s represents the speed of sound (=330 m/s) and v represents the speed of the airplane.



D1. Find the speed of the plane.

(1.75 points)

Problem 2 – An experimental analysis of star luminosity - 10.00 points)

It is a well-known fact that stars shine. What does this mean? It means they emit light. Light is a form of electromagnetic energy, so, as for any other form of energy, we can define a power (the amount of energy radiated in a unit of time—measured in watts). In the case of stars, we call this power luminosity and denote it by L. We also define the luminosity per unit area of the star as $M = \frac{L}{4\pi R^2}$, where R is the radius of the star.

The angular diameter of a star is defined as the angle subtended by the star in a circle centered on the Earth. We usually measure it in milliarcseconds (mas), where one arcsecond is $\frac{1}{3600}$ of a degree.

Interstellar distances are measured in parsecs (pc), defined by $1\ \text{pc} = 3.1 \cdot 10^{16}$ m. The following table with data is given for some stars (the data has been slightly changed to fit the model presented in this problem):

Star	δ (mas)	d (pc)	T (K)	f ($\frac{W}{m^2}$)
Arcturus	21.0	11.26	4290	$4.91 \cdot 10^{-8}$
Vega	3.28	7.68	9600	$3.09 \cdot 10^{-8}$
Sirius A	6.04	2.64	9845	$1.14 \cdot 10^{-7}$
Dubhe	3.0	37.7	4700	$1.51 \cdot 10^{-9}$
Procyon	5.4	3.51	6500	$1.81 \cdot 10^{-8}$

Where δ is the angular diameter, d is the distance from us to the star, T is the absolute temperature of the star and f is the energy per unit area measured by an observer on Earth.

A. Let's consider a star with luminous flux measured from the Earth f, with a radius R and found at a distance d from the Earth. Find a formula for the luminosity per unit area of the star M, using the conservation of energy and assuming a spherically symmetric distribution of the energy.

(1.50 points)

B. Fill in the following table:

Star	δ (rad)	d (m)	R (m)	M $\left(\frac{W}{m^2}\right)$
Arcturus				
Vega				
Sirius A				
Dubhe				
Procyon				

(3.50 points)

C. The luminosity per unit area of the star is given by the Stefan-Boltzmann law, $M = \sigma T^n$, where n is a real number and σ is called the Stefan-Boltzmann constant. By taking the natural logarithm of this equation, we get $\ln M = \ln \sigma + n \cdot \ln T$.

On a sheet of graph paper, graph ln M as a function of ln T.

(2.00 points)

D. Using the graph, find the value of n.

(1.50 points)

E. Using the found value of n, calculate the value of σ for each of the stars. Calculate the mean value.

(1.50 points)

Don't forget that every experimental data has a certain amount of error, so the values determined might not match the ones you know.

Problem 3 – Climate Physics - 10.00 points

Clouds are visible collections of water droplets suspended in the sky that form due to the sharp drops in temperature.

Part A – Modeling cloud formation - 4.80 points

A1. What phenomenon takes place when clouds form?

- ☐ Vaporization
- ☐ Sublimation
- ☐ Condensation
- ☐ De-sublimation (= deposition)

(0.20 points)

A2. The temperature decreases approximately linearly when altitude gets higher, at a rate approximately equal to $\Gamma = 6.5°C/km = 6.5\ K/km$. Knowing that at sea-level the temperature is T_0 = 290K, fill the following table:

h/km	T/K
0.5	
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	

(0.70 points)

The atmospheric pressure is equal to $P_0 = 1.0 \times 10^5 Pa$ at sea-level and decreases exponentially with height: $P(h) = P_0 e^{-\frac{Mg}{RT_0}h}$

In the given formula, M is the average molar mass of air ($M \approx 2.89 \times 10^{-2} kg/mol$), g is the acceleration due to gravity, R is the ideal gas constant and T_0 is the temperature at sea-level. The mole fraction of water vapor in the atmosphere also decreases approximately exponentially from $x_0 = 1.5\%$ at sea-level, according to:

$$x(h) = x_0 e^{-\frac{h}{H}}$$

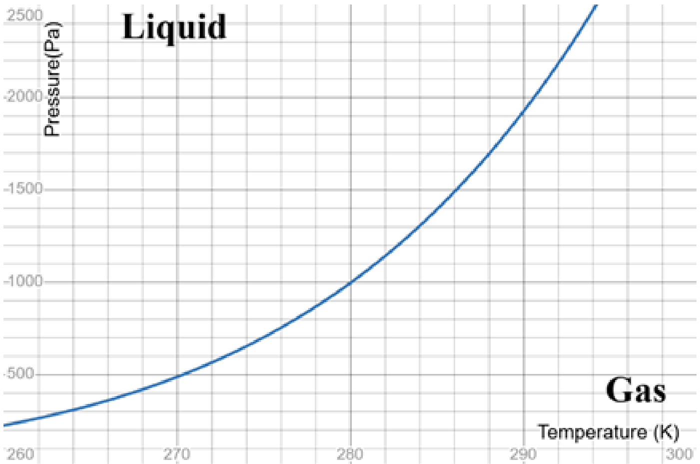
Where H is a parameter determined experimentally to be $H \approx 4.1 \times 10^3 m$.

A3. Fill the following table with values of the partial pressure of water, knowing that the partial pressure of a gas in a mixture is directly proportional to its mole fraction:

h/km	P_{H_2O} / Pa
0.0	
0.5	
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	

(1.60 points)

The graph below gives a small portion of the phase diagram of water. A larger version is provided on the answer sheet.



A4. On the phase diagram, plot each of the found (T, P_{H2O}) points. It might be useful to fill the following table (no points will be deducted for not filling it out):

h/km	T/K	P_{H_2O} / Pa
0.0		
0.5		
1.0		
1.5		
2.0		
2.5		
3.0		
3.5		

(1.20 points)

A5. On the phase diagram, trace a curve going through all the points drawn in part A4.

(0.50 points)

A6. What is the temperature at the height where clouds are expected to form? Using that, at what height do you expect clouds to form?

(0.60 points)

The provided model uses a few inaccurate assumptions; however, the values are quite realistic, but clouds form in a large range of heights, due to phenomena more complex than those described in this problem.

Part B – Cloud electrification - 2.90 points

During thunderstorms, clouds become charged, making it possible for lighting strikes to happen. We will study the different ways clouds can get electrized (friction and contact).

B1. Consider two neutral clouds, after a frictional contact between them, they will end up being charged, what type of charge will each cloud have?

- ☐ Both Positive
- ☐ Both Negative
- ☐ One positive and one negative

(0.30 points)

Consider now, another two identical clouds (1 and 2) in shape, both have a volume of 200 m^3. 10% of the volume of Cloud 1 is positively charged, with charge density of 5 C/m^3, while the other one (Cloud 2) is neutral. A contact electrification is established between them.

(Consider the clouds to be approximately spherical)

B2. Calculate the new charge of cloud 1 and the new charge of cloud 2.

(0.60 points)

Electrically charged clouds can also induce a potential difference with the ground, consider that cloud 1 is 5 km above the ground, determine.

(If you didn't solve B2, consider the charge of cloud 1 to be 80 C)

For B4, B5, B6 and B7, consider the clouds to be point-like.

B3. The electric field due to cloud 1 on the ground, in a point right below it.

(0.85 points)

B4. The electric potential due to cloud 1 on the same point.

(0.85 points)

B5. Despite the potential difference between a cloud and the ground, we cannot assure that a lightning strike will actually occur, what is the best explanation for that?

- ☐ The potential difference is very small
- ☐ The air acts as an insulator, making the flow of electric charge difficult
- ☐ There needs to be a highly charged tall object on the ground
- ☐ Clouds quickly go through a discharge.

(0.30 points)

Part C – A different model for thunderstorms - 2.30 points

Instead of a point-like object, let's now consider that a cloud is a perfectly conductive circular disk of radius R = 5 km, still found at H = 5 km.

C1. Assuming that air has the same electrical permittivity as vacuum, find the capacitance of the system formed by the cloud and the ground under it.

If you couldn't solve C1, use C = 1μF

(0.50 points)

C2. If the cloud is charged with Q = +100C, find the potential difference between the ground and the cloud.

(0.50 points)

C3. A lightning strike can be modeled as an electrical current which increases linearly from 0 A to $2 \cdot 10^6$ A in the span of 50 μs, and then abruptly decreases to 0. After a lightning strike, what value will the new potential difference between the ground and the cloud have?

(1.30 points)

—End of the Paper—